NUCLEON ELECTROMAGNETIC FORM FACTORS IN THE PERTURBATIVE CHIRAL QUARK MODEL

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2019

ฟอร์มแฟคเตอร์แม่เหล็กไฟฟ้าของนิวคลีออนในแบบจำลองเพอร์เทอร์เบทีฟไครอลควาร์ก

จรูญศักดิ์ จรัสศรีวิไล

ปริญญานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตร
ปรัชญาดุษฎีบัณฑิต สาขาวิชาฟิสิกส์
คณะวิทยาศาสตร์ มหาวิทยาลัยศรีนครินทรวิโรฒ
ปีการศึกษา 2562

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A Dissertation Submitted in Partial Fulfillment of the Requirements

for the Degree of DOCTOR OF PHILOSOPHY

(Physics)

Faculty of Science, Srinakharinwirot University

2019

THE DISSERTATION TITLED

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BY

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HAS BEEN APPROVED BY THE GRADUATE SCHOOL IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DOCTOR OF PHILOSOPHY
IN PHYSICS AT SRINAKHARINWIROT UNIVERSITY

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Title	NUCLEON ELECTROMAGNETIC FORM FACTORS IN THE PERTURBATIVE CHIRAL QUARK MODEL
Author	JAROONSAK JARASSRIWILAI
Degree	DOCTOR OF PHILOSOPHY
Academic Year	2019
Thesis Advisor	Dr. Kem Pumsa-ard
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The electromagnetic form factors of the nucleon, the common name for the proton and the neutron, are fundamental properties that play an essential role in the study of the internal structure of the nucleon. In this work, the electromagnetic properties of the nucleon were calculated using the Perturbative Chiral Quark Model (PCQM). In the PCQM, the nucleon is considered to be the bound state of the three valence quarks surrounded by the cloud of the Goldstone bosons: the pions, kaons and eta meson. Previously, the electromagnetic properties were studied based on this model and using the truncated quark propagator, restricted only to the quark in the ground state. An attempt to include the excited states quark propagators in the study of the nucleon electromagnetic form factors was performed with this model; however, it did apply to the case of the neutron, but not the proton. Therefore, to bring the consistency to the nucleonic level, the study was extended by including the second excited state quark propagators to calculate the electromagnetic form factors and applied to both the proton and the neutron. The results were in good agreement with the experimental data and showed the significance of the contributions of the quark excited state propagators to the electromagnetic properties of the nucleon.

Keyword : Nucleon electromagnetic form factors, Perturbative chiral quark model

ACKNOWLEDGEMENTS

I would like to convery many thanks to Dr. Kem Pumsa-ard for being my thesis major advisor and for his very good and fruitful guidance throughout this work.

And also, I would like to thank Assist. Prof. Dr. Patipan Utayarat for good guidance and helpful advice as my thesis co-advisor of this work.

Finally, I would especially like to thank my parents,Jia Saeton and Ju Saeton, my beloved brothers, Charoen , Chamrat , Charun and sisters, Ornsiri, Prapin, Chitra and my good friends for their understanding, support and encouragement over the years of my study.

JAROONSAK JARASSRIWILAI

TABLE OF CONTENTS

Page

LIST OF TABLE

Page

LIST OF FIGURE

Page

CHAPTER 1
INTRODUCTION

Nucleon, a collective name for proton and neutron, is an important particle in Particle Physics. Understanding the internal structure of the nucleons and their properties are fundamental importance. However, the ultimate understanding of the nucleon has not been achieved yet. Both theoretical and advanced experimental works at ELSA, JLab, MAMI, MIT, NIKHEF and other laboratories are still ongoing in order to reach that goal. Up to date, there is no theoretical framework that can thoroughly describe the nucleon electromagnetic properties.

One of the difficulties comes from the fact that studying the internal structure of nucleon by directly using the fundamental theory–the Quantum Chromodynamics (QCD) is extremely difficult and needs many complicated diagrams. Meanwhile, the other approaches, such as phenomenological models, or formulation of QCD on the lattice (lattice QCD) are options for studying nucleon properties but still needs higher performance computation. The complicated calculations of QCD in the non-perturbative region, or at low four-momentum transfer, raised a necessary on development of other models, such as the effective field theory (EFT). It was around the early of eighties a new model has been introduced, the so-called chiral quark model (1). Originally, this idea was formulated in the context of the cloudy bag model in which the nucleon is treated as a bound system of valence quarks with a surrounding meson cloud. So far, this chiral quark model has been researching and developing continually, and up to now, this model plays an important role in low-energy physics.

In 2001, Lyubovitskij et al. (1) proposed an extended relativistic quark model, which not only improved the calculation results when comparing with experimental data but also be able to fulfill many chiral constraints. The Goldstone bosons are introduced as a consequence of chiral symmetry breaking. In the study, the core improvements of the proposed model were mainly i) starting the SU(3), i.e., the meson cloud composes of the π, K and η mesons which are treated perturbatively; ii) consistency of perturbation theory for both on the quark and nucleonic level by use of renormalization methods; iii) allowing to account for excited quark states in the meson loop diagrams; iv) fulfillment of the constraints imposed by chiral symmetry, including the current quark mass in the nucleon. Furthermore, the sea-quark contributions were already included in the Goldstone mesons cloud effect of this model, so-called the Perturbative Chiral Quark Model (PCQM).

In this study, we use the PCQM in order to calculate the electromagnetic properties of the nucleon, i.e., the magnetic moments, charge radii, magnetization radii of both proton and neutron. Besides those properties, we also investigate the four-momentum transfer Q^2-dependent electric form factors $G_E^N(Q^2)$ and magnetic form factor $G_M^N(Q^2)$ of the nucleon. The slope of the electric form factor $G_E^N(Q^2)$and magnetic form factors $G_M^N(Q^2)$ at zero momentum ($Q^2 = 0$) provides the electric and magnetic root mean square radius value, while the values of the electric and magnetic form factors at zero momentum give the electric charge and magnetic moments of the nucleon. The effective Lagrangian of the PCQM is formulated to describe quarks moving in a radially quadratic effective potential $V_{eff} = S(r) + \gamma^0 V(r)$ with $r = |\vec{x}|$. We determined the scalar potential $S(r)$ as a potential which is responsible for the confinement of quarks and the vector potential $V(r)$ as a potential which is responsible for short-range effects of the gluon field. From the results of previous works (1, 2), we may consider the mesons fields as small fluctuations to the system, makes us be able to restrict our calculations only to the linear form of the strong meson-quark interaction, is given by

$$\mathcal{L}_{int}^{str} = -\bar{\psi}(x)S(r)i\gamma^5 \frac{\hat{\Phi}(x)}{F}\,\psi(x) \tag{1.1}$$

For our perturbative scheme, we calculate the meson loop diagram restricted to only one loop, or up to the order of $\mathcal{O}\left(\frac{1}{F^2}\right)$. We also treat the mass term of the valence quarks as a small perturbation to the system.

There were calculations reported in Ref. (1, 3-6) on the Electromagnetic form factors of the nucleon. In Ref. (2) the quark propagator with the excited quark states is included in the calculational technique of the PCQM. However, only applied the charge form factor of the neutron. It is well known that the three valence quarks give zero contribution to the charge form factor of the neutron, and the main contribution comes from the meson contributions.

The valence quarks inside nucleons are structureless and spin-1/2 particle. Therefore in our model, they are described by Dirac wave function $\psi_\alpha(x)$ of any state of α including ground state $\alpha = 0$ and excited state $\alpha = 1,2,..$, where $x \equiv x^\mu = (t, x, y, z)$,

$$\psi_\alpha(x) = b_0 u_\alpha(\vec{x})\, exp(-i\mathcal{E}_\alpha t). \tag{1.2}$$

The wave function $u_\alpha(\vec{x})$ is the solution of the Dirac equation with a general quadratic form of confinement potential,

$$\left[-i\vec{\alpha}\cdot\vec{\nabla} + \beta S(r) + V(r) - \mathcal{E}_\alpha\right]u_\alpha(\vec{x}) = 0. \tag{1.3}$$

For the quadratic-like potential $S(r)$ and $V(r)$, the explicit form of $u_\alpha(\vec{x})$ is

$$u_\alpha(\vec{x}) = N_\alpha\, exp\left(-\frac{\vec{x}^2}{2R^2}\right)\begin{pmatrix} g_\alpha(r) \\ i\vec{\sigma}\cdot\hat{x} f_\alpha(r)\end{pmatrix}\mathcal{Y}_\alpha(\hat{x})\chi_f\chi_c. \tag{1.4}$$

The $g_\alpha(r)$ and $f_\alpha(r)$ are the upper and lower components of the quark spinor, respectively. It means that we have two free parameters in our quark wave function. We know that these two parameters are important factors in shaping the quark wave function, which can affect our results. Further, because they are related to energy $\mathcal{E}_\alpha$ of the quark propagator, so values of the parameter should vary with respect to the number of excited states we consider in the model. Here we define $\Delta\mathcal{E}_\alpha = \mathcal{E}_\alpha - \mathcal{E}_0$ to be the difference between the energy of excited state α, denoted by $\mathcal{E}_\alpha$ and the ground state energy, denoted by $\mathcal{E}_0$. The important relation for $\Delta\mathcal{E}_\alpha$, in terms of the quantum numbers n and l, is

$$\left(\Delta\mathcal{E}_\alpha + \frac{3\rho}{R}\right)^2\left(\Delta\mathcal{E}_\alpha + \frac{1}{\rho R}\right) = \frac{\rho}{R^3}(4n + 2l - 1)^2. \tag{1.5}$$

In phenomenological methods, using an appropriate set of parameters can provide good calculation results when comparing with experimental data and also reveal some hidden information.

In this work, we will base on the PCQM in order to calculate electromagnetic form factors and properties of the nucleon by using the modified quark propagator.

Research objectives

1. To modify the model by using the quark propagator in the first and second excited states and use this quark propagator to calculate the electromagnetic form factors of the nucleon.

2. To fit the static properties such as the magnetic moments, charged radius and magnetic radius of nucleon by using the appropriate values of the model parameters ρ and R extracted from the experimental data..

Significance of research

For consistency, in this work, the excited state of quarks will be included to both proton and neutron to study the electromagnetic form factors of nucleon. Therefore, the contributions to the electromagnetic properties of nucleon by the quark excited state in the propagator will be clarified.

Scope of research

1. In this work, the order of accuracy of the PCQM up to the order of $\mathcal{O}\left(\frac{1}{F^2}, \hat{m}, m_s\right)$ will be considered.

2. We will restrict ourselves to the case where the low-lying quark excited states are included in the quark propagator.

3. Only the small four-momentum squared region up to $0.4\ \mathrm{GeV}^2$ will be considered ($Q^2 \leq 0.4\ \mathrm{GeV}^2$).).

CHAPTER 2
LITERATURE REVIEW

Recently, there has been a lot of theoretical researches and experiments in the field of the nucleon and other baryon octet focused on electromagnetic properties, in order to understand the internal structures of those particles. In decades, the theoretical description of electromagnetic form factors was performed in detail by approaches of hadron physics, such as QCD Sum Rules, Chiral Perturbation Theory, Lattice QCD, and AdS/QCD technique. In this chapter, we raise some essential works which are concerning our model of study and some of the works which are recently in the area of interest from hadronic particle physicists.

In 2001, Kubis et at. used the chiral expansion method in order to analyze the electromagnetic form factors of the baryon in (3). The effective Lagrangian of the system consists of a string of terms of increasing chiral dimension. In the study, they analyzed internal structures of the baryon by probing with four-momentum transfer $Q^2 \approx 0.4$ GeV^2. The effective Lagrangian of the model is given below.

$$\mathcal{L}_{eff} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)} + \cdots. \qquad (2.1)$$

Where the ellipsis denotes terms of higher-order not needed. The chiral effective pion-pion Lagrangian is given by

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle, \qquad (2.2)$$

The pion–nucleon Lagrangian at leading order is

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}\left(i\gamma^\mu D - m + \frac{g_A}{2}\gamma^\mu u_\mu \gamma_5\right)\Psi, \qquad (2.3)$$

They evaluated the electromagnetic form factors in term of matrix element which is described in terms of both the Dirac form factor $F_1^N(Q^2)$and Pauli form factor $F_2^N(Q^2)$ as,

$$\langle N(p_f)|J^\mu(0)|N(p_i)\rangle = \bar{u}(p_f)\left[\gamma_\mu F_1^N(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2^N(Q^2)\right]u(p_i). \qquad (2.5)$$

and also in terms of the electric and magnetic Sachs form factors $G_E^N(Q^2)$ and $G_M^N(Q^2)$ which defined by

$$G_E^N(Q^2) = F_1^N(Q^2) - \frac{Q^2}{4m_N^2} F_2^N(Q^2),$$
$$G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2). \quad (2.6)$$

As a result, they could perform the calculations, which gave a good description of the nucleon charge form factor. Even though the results still needed some improvement, but the results had shown the consistency of the effective chiral perturbation method with the experimental data.

Furthermore, they extended the study to investigate the octet baryons form factors in (4). They changed the SU(2) calculation to be the SU(3) case. In the theoretical calculation, they spelled out the effective chiral Lagrangian into two parts, the meson–baryon Lagrangian and the chiral effective Lagrangian from Goldstone bosons. The meson–baryon Lagrangian at leading order was

$$\mathcal{L}_{\phi B}^{(1)} = \langle \bar{B}(i\gamma^\mu D - m)B \rangle + \frac{D}{2F} \langle \bar{B}\gamma^\mu \gamma_5 (u_\mu, B)_\pm \rangle. \quad (2.7)$$

And the chiral effective Goldstone boson Lagrangian was given by

$$\mathcal{L}_{\phi\phi}^{(2)} = \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle. \quad (2.8)$$

They evaluated the electromagnetic properties of the nucleon and also predicted the charge and magnetic radii of the octet baryons, together with their magnetic moments which still had no experiment data yet, for example, Λ and Σ^+ . In the results, they found those properties, including magnetic moments and electric radii, show the same tendency to the experimental data. Nevertheless, the magnetic radii still needed some improvement.

In 2004, Fuchs et al. calculated the electromagnetic form factor of nucleon at q^4 and made use of the extended on-mass-shell renormalization method (5). They evaluated the electromagnetic form factors in term of matrix element which is described in terms of both the Dirac form factor $F_1^N(Q^2)$ and the Pauli form factor $F_2^N(Q^2)$ and in terms of the electric and magnetic Sachs form actors $G_E^N(Q^2)$ and $G_M^N(Q^2)$. The Fourier transforms of the Sachs form factors can be related to the distribution of charge and magnetization inside the nucleon. They compared the results with those obtained in the heavy-baryon approach and in the infrared regularization and found the results were almost similar. However, there were small differences between the two methods, due to the way how the regular higher-order terms of loop integrals were treated.

For further improvement, in 2005, Matthias R. Schindler included the vector mesons as explicit degrees of freedom into the model of study (6). The coupling of the vector mesons to pions and external fields is given by

$$\mathcal{L}_{\pi V}^{(3)} = -f_\rho \, Tr.\left(\rho^{\mu\nu} f_{\mu\nu}^{+}\right) - f_\omega \omega^{\mu\nu} f_{\mu\nu}^{(s)} - f_\phi \phi^{\mu\nu} f_{\mu\nu}^{(s)} + \cdots. \tag{2.9}$$

They found the results for the Sachs form factors in the low momentum transfer region $0 \text{ GeV}^2 \leq \text{Q}^2 \leq 0.4\text{GeV}^2$ with vector mesons were considerably improved. Moreover, the most dominant contributions to electromagnetic form factors came from tree-level diagrams, while loop corrections with internal vector meson contributions were small.

In 2018, Thomas Gutsche, Valery E. Lyubovitskij, and Ivan Schmidt reported calculation results of electromagnetic properties of the nucleon and the Roper resonance based on the AdS/QCD (7). In the study, they described the system by conformal Poincar´e metric,

$$\begin{aligned} g_{MN} x^M x^N &= \epsilon_M^a \epsilon_N^a \eta_{ab} x^M x^N) \\ &= \frac{1}{z^2}(dx_\mu dx^\mu - dz^2 \end{aligned} \tag{2.10}$$

The action S which is a temperature-dependent (T), contains a free part S_0, describing the confined dynamics of AdS fields, and an interaction part S_{int}, describing the interactions of fermions with the vector field.

$$\begin{aligned} S &= S_0 + S_{int}, \\ S_0 &= \int d^4xdz \sqrt{g} e^{-\varphi(z,T)} \{\mathcal{L}_N(x,z,T) + \mathcal{L}_R(x,z,T) + \mathcal{L}_V(x,z,T)\}, \\ S_{int} &= \int d^4xdz \sqrt{g} e^{-\varphi(z,T)} \{\mathcal{L}_{VNN}(x,z) + \mathcal{L}_{VRR}(x,z,T) \\ &\qquad + \mathcal{L}_{VRN}(x,z,T)\}. \end{aligned} \tag{2.11}$$

The definitions of the temperature-dependent nucleon Sachs form factors $G_{E,M}^N(Q^2,T)$ the temperature-dependent electromagnetic radii $\langle r_{E,M}^2(T)\rangle^N$ in terms of the Dirac form factor $F_1^N(Q^2,T)$ and Pauli form factor $F_2^N(Q^2,T)$ are shown below

$$\begin{aligned} G_E^N(Q^2,T) &= F_1^N(Q^2,T) - \frac{Q^2}{4M_N^2(T)} F_2^N(Q^2,T), \\ G_M^N(Q^2,T) &= F_1^N(Q^2,T) + F_2^N(Q^2,T) \end{aligned} \tag{2.12}$$

$$\langle r_E^2(T)\rangle^N = -6\frac{dG_E^N(Q^2,T)}{dQ^2}\bigg|_{Q^2=0}$$

$$\langle r_M^2(T)\rangle^N = -\frac{6}{G_M^N(0)}\frac{dG_M^N(Q^2,T)}{dQ^2}\bigg|_{Q^2=0}.$$

Recently, lattice QCD is becoming one of the powerful approaches to investigate the properties of nucleons. However, it still needs some higher performance in a computer simulation. In 2017, C. Alexandrou et al., performed the calculations by using the lattice QCD method [6]. In their work, they used three methods to extract information on nucleon electromagnetic properties from the lattice data. The three methods were 1) Plateau method, 2) the Two-state fit method and 3) Summation method. The simulation parameters and results of calculations such as isovector and isoscalar contribution at physical pion mass of around 130 MeV were evaluated. Also, the Sachs nucleon form factors, including the disconnected contributions, were presented directly at the physical point. In the study, the Isovector and isoscalar form factors data were compared with the dipole fits as below.

$$G_i(Q^2) = \frac{G_i(0)}{(1+\frac{Q^2}{M_i^2})^2}. \tag{2.13}$$

The PCQM is another approach to investigate the internal structures of nucleons for more than two decades. Application of the PCQM to study various baryons properties are reported, e.g., the πN σ-terms (8, 9). The the πN σ-terms are known as the fundamental parameters of low-energy hadron physics since they provide a direct measure of the scalar quark condensates in nucleons. In particular, the sigma-terms are mostly determined by the quark-antiquark sea and not by the valence quark contribution. The PCQM will be mentioned in more detail in chapter 3. In this model, the cloud of virtual Goldstone mesons that surrounds any baryon contributes to the mass and other properties of that particle. As a result, the sigma-term $\sigma_{\pi N}$ with the quark propagator restricted to the ground state was calculated in (9).

In 2001, V. E. Lyubovitskij et al. applied the PCQM to evaluate analytical results for the nucleon charge and magnetic form factors (1). The model was based on an effective Lagrangian, where nucleons were described by relativistic valence quarks surrounded by a perturbative cloud of Goldstone bosons. The Lagrangian of the system was given by

$$\mathcal{L}_{eff} = \mathcal{L}_{inv}^{lin} + \mathcal{L}_{\chi SB}. \tag{2.14}$$

Where,

$$\begin{aligned}\mathcal{L}_{inv}^{lin}(x) &= \bar{\psi}(x)\left[i\gamma^{\mu}\partial_{\mu} - S(r) - \gamma^{0}V(r)\right]\psi(x) + \frac{1}{2}(\partial_{\mu}\hat{\Phi}(x))^{2} \\ &\quad - \bar{\psi}(x)S(r)i\gamma^{5}\frac{\hat{\Phi}(x)}{F}\psi(x), \\ \mathcal{L}_{\chi SB} &= -\bar{\psi}(x)\mathcal{M}\psi(x) - \frac{B}{2}Tr\left[\hat{\Phi}^{2}(x)\mathcal{M}\right].\end{aligned} \tag{2.15}$$

The quark wave function restricted to the ground state was given below.

$$\psi(x) = b_0 u_0(\vec{x}) exp(-i\varepsilon_0 t), \tag{2.16}$$

The wave function $u_0(\vec{x})$ belongs to the basis of potential eigenstates. $S(r)$is the scalar confinement potential and $V(r)$ is the vector potential of the model. Both of them are quadratic radial dependence.

$$\begin{aligned}S(r) &= M_1 + c_1 r^2, \\ V(r) &= M_2 + c_2 r^2.\end{aligned} \tag{2.17}$$

Furthermore, for the sake of simplicity and be consistent with the potential model, they introduced a Gaussian ansatz as the quark wave function with the explicit form as

$$u_0(\vec{x}) = N_0\, exp\left(-\frac{\vec{x}^2}{2R^2}\right)\begin{pmatrix}1 \\ i\rho\frac{\vec{\sigma}\cdot\vec{x}}{R}\end{pmatrix}\chi_s\chi_f\chi_c. \tag{2.18}$$

Besides, they introduced a nucleon charge and quark mass renormalization into the calculation, in order to: i) to maintain the proper definition of physical parameters, such as nucleon mass and, in particular, the nucleon charge and ii) to effectively reduce the number of Feynman diagrams to be evaluated. Then the renormalized quark field becomes as below.

$$\begin{aligned}\psi_i^r(x; m_i^r) &= b_0 u_0^r(\vec{x}; m_i^r) exp(-i\varepsilon_0^r(m_i^r)t), \\ u_0^r(\vec{x}; m_i^r) &= N_0\,(m_i^r) exp\left(-c(m_i^r)\frac{\vec{x}^2}{2R^2}\right)\begin{pmatrix}1 \\ i\rho(m_i^r)\frac{\vec{\sigma}\cdot\vec{x}}{R}\end{pmatrix}\chi_s\chi_f\chi_c.\end{aligned} \tag{2.19}$$

In the results, they found the most of the electromagnetic properties, such as magnetic moments, proton charge radius and magnetic radii, except the neutron charge radius, were in the same tendency of experimental data. Even though the magnitude value of the neutron charge radius was too small, but the sign was correct.

In 2004, in order to investigate the internal structure of other baryon octets besides nucleon, S. Cheedket et al. applied the PCQM to evaluate the electromagnetic properties of the baryon octet (2). They evaluated the electromagnetic form factors of the baryon in the Breit frame, where gauge invariance is fulfilled. The Sachs charge form factor G_E^B and magnetic form factors G_M^B of the baryon in term of momentum transfer Q^2as below.

$$\begin{aligned} &\left\langle B_{\acute{s}}\left(\frac{\vec{q}}{2}\right)\middle| J^0(0) \middle| B_s\left(-\frac{\vec{q}}{2}\right)\right\rangle = \chi^\dagger_{B_{\acute{s}}}\chi_{B_s} G_E^B(Q^2), \\ &\left\langle B_{\acute{s}}\left(\frac{\vec{q}}{2}\right)\middle| \vec{J}(0) \middle| B_s\left(-\frac{\vec{q}}{2}\right)\right\rangle = \chi^\dagger_{B_{\acute{s}}}\, i\frac{\vec{\sigma}_N\times\vec{q}}{2m_B}\chi_{B_s} G_M^B(Q^2). \end{aligned} \tag{2.20}$$

Expressions for the Sachs form factors in the PCQM are

$$\begin{aligned} &\chi^\dagger_{\acute{s}}\chi_s G_E^B(Q^2) = \\ &\left\langle \phi_0 \middle| \sum_{n=0}^{2}\frac{i^n}{n!}\int \delta(t)\, d^4x d^4x_1 \dots d^4x_n e^{-iq\cdot x} T[\mathcal{L}_r^{str}(x_1)\dots\mathcal{L}_r^{str}(x_n) J_r^0(x)] \middle| \phi_0 \right\rangle, \end{aligned} \tag{2.21}$$

and

$$\begin{aligned} &\chi^\dagger_{\acute{s}}\, i\frac{\vec{\sigma}_B\times\vec{q}}{m_B+m_{\acute{B}}}\chi_{B_s} G_M^B(Q^2) = \\ &\left\langle \phi_0 \middle| \sum_{n=0}^{2}\frac{i^n}{n!}\int \delta(t)\, d^4x d^4x_1 \dots d^4x_n e^{-iq\cdot x} T[\mathcal{L}_r^{str}(x_1)\dots\mathcal{L}_r^{str}(x_n) \vec{J}_r(x)] \middle| \phi_0 \right\rangle. \end{aligned} \tag{2.22}$$

Note that, in this study, the modification of the quark propagator is investigated only to the charge neutron form factor. Nevertheless, it was shown that the excited states quark propagator, together with effect from the meson cloud, help to improve the values of the low-Q^2 region of the charge neutron form factors.

Furthermore, in (10) Pumsa-ard et al. applied PCQM to investigate the electromagnetic transitions of the nucleon to baryon excitations properties, which provide important information about hadron physics and the structure of the nucleon. In the study, they considered the determination of the momentum dependence of the helicity amplitudes $A_{1/2}$ and $A_{3/2}$ of the $\mathrm{N}-\Delta$ transition at one-loop, and also investigated the role of excited quark states in meson loop diagrams. In the PCQM the helicity amplitudes $A_{1/2}$ and $A_{3/2}$, are defined as below.

$$A_{\frac{1}{2}}(Q^2) = -\frac{e}{\sqrt{2\omega_\gamma}} < \Delta^+, 1/2 \left| -\frac{1}{2} \int \delta(t) d^4x d^4x_1 d^4x_2 e^{-iqx} \right.$$
$$\times T[\mathcal{L}^r_{str}(x_1)\mathcal{L}^r_{str}(x_2)\vec{J_r}(x) \cdot \vec{\epsilon}]|p, -1/2 >_c, \quad (2.23)$$
$$A_{\frac{3}{2}}(Q^2) = -\frac{e}{\sqrt{2\omega_\gamma}} < \Delta^+, 3/2 \left| -\frac{1}{2} \int \delta(t) d^4x d^4x_1 d^4x_2 e^{-iqx} \right.$$
$$\times T[\mathcal{L}^r_{str}(x_1)\mathcal{L}^r_{str}(x_2)\vec{J_r}(x) \cdot \vec{\epsilon}]|p, 1/2 >_c$$

As a result, they accomplished in calculations numerically of the transverse helicity amplitudes for the $\mathrm{N} - \Delta$ transition by the PCQM approach. Besides, they could also showed the importance of meson cloud corrections related to the magnitude of the helicity amplitudes. They found that the excited quark states could contribute significantly at the level of 15% to fully account for the measurements.

The strange quark contributions to the properties of the nucleon also have been studied and reported in (11). They evaluated both of the strange vector and axial-vector nucleon form factors and seemed to be consistent with the SAMPLE and HAPPEX experimental data. The definitions of nucleon form factors were shown below

$$G^s_E(Q^2) = F^s_1(Q^2) - \frac{Q^2}{4m^2_N} F^s_2(Q^2), \quad (2.24)$$
$$G^s_M(Q^2) = F^s_1(Q^2) + F^s_2(Q^2).$$

Besides, in 2006 the PCQM was used to study the nucleon spin-independent polarizabilities(α_E and β_M) by Y. Dong et al. (12). The results of including excited quarks states were calculated and compared with the excluded ones.

So far, most previous calculations have been truncated to the use of the ground state quark propagator. Recently, the progress of improving the charge form factor of the charge neutron in the PCQM has been performed with the modified quark propagator but with different potentials (13). X. Y. Liu used the PCQM to investigate the charge form factor and charge radius of the neutron with considering both the ground and excited states in the quark propagator. In their work, they introduced the Cornell-like potential and solved the Dirac equation to get the ground state quark wave function, and the excited quark states. Due to the scope of the study, in the calculation, they restricted the energy of excited quark states to $\mathrm{E}_\alpha = 1\,\mathrm{GeV}$, which are $1p_{1/2}, 1p_{3/2}, 1d_{3/2}, 1d_{5/2}, 1f_{5/2}, 1f_{7/2}, 2s_{1/2}, 2p_{1/2}, 2p_{3/2}$ and $3s_{1/2}$, while meson cloud contributions were from π-meson cloud only. The interaction Lagrangian of the model is given below,

$$\mathcal{L}_I^W(x) = \frac{1}{2F}\partial_\mu \Phi_i(x)\bar{\psi}(x)\gamma^\mu\gamma^5\lambda_i\psi(x). \tag{2.25}$$

where $F = 88MeV$ is the so-called the π-decay constant. Four types of diagrams contribute to the charge form factor of the neutron, which are (i) meson cloud diagram, (ii) vertex correction diagram, (iii) self-energy I diagram, and (iv) self-energy II diagram. Furthermore, the charge current of the system can be divided into three pieces, which are the quark charge current, meson charge current, and the interacting term between quark and meson. The total charge current j_0 is given by,

$$\begin{aligned} j_0 = \bar{\psi}\gamma^0 Q\psi + \left[f_{3ij} + \frac{f_{8ij}}{\sqrt{3}}\right]\Phi_i(x)\partial_t\Phi_j(x) \\ + \left[f_{3ij} + \frac{f_{8ij}}{\sqrt{3}}\right]\frac{\Phi_j}{2F}\bar{\psi}\gamma^0\gamma^5\lambda_i\psi. \end{aligned} \tag{2.26}$$

In the results, for the low four-momentum transfer Q^2range, they found that the excited quark states were considerably influential in the charge form factor and charge radius of the neutron. The total result of the neutron charge radius $\langle r_E^2\rangle^n$ increases from -0.014 to be -0.072 when the excited states are included.

Besides the study of nucleon properties through the Sachs form factor, there are also many kinds of research through the axial form factor approach. Those systems are described by the Weinberg-type form, containing the axial-vector coupling. In 2004, K Khosonthongkee et al. applied the PCQM to study the axial form factor of the nucleon (14). The axial form factor is one of the fundamental weak interaction properties. In the study, they have used the axial-vector coupling Lagrangian $\mathcal{L}^W(x)$ of the Weinberg-type,

$$\begin{aligned} \mathcal{L}^W(x) &= \mathcal{L}_0(x) + \mathcal{L}_I^W + \mathcal{O}(\pi^2), \\ \mathcal{L}_0(x) &= \bar{\psi}(x)\{(i\gamma^\mu - S(r) - \gamma^0 V(r)\}\psi(x) - \frac{1}{2}\pi(x)(\square + M_\pi^2)\pi(x). \end{aligned} \tag{2.27}$$

Where, the strong interaction Lagrangian, $\mathcal{L}_{I,str}^W$ is given by

$$\begin{aligned} \mathcal{L}_{I,str}^W(x) = \frac{1}{2F}\partial_\mu\pi(x)\bar{\psi}(x)\gamma^\mu\gamma^5\tau\psi(x) \\ -\frac{\varepsilon_{ijk}}{4F^2}\pi_i(x)\partial_\mu\pi_j(x)\bar{\psi}(x)\gamma^\mu\tau_k\psi(x). \end{aligned} \tag{2.28}$$

And the interaction between pions and quarks by the electromagnetic field $\mathcal{L}_{I,em}^W(x)$ is given by,

$$\mathcal{L}_{I,em}^{W}(x) = -eA_{\mu}^{em}\bar{\psi}^{r}(x)Q\gamma^{\mu}\psi^{r}(x)$$
$$+\frac{e}{4F^2}A_{\mu}^{em}\bar{\psi}^{r}(x)\gamma^{\mu}[\pi^2(x)\tau_3 - \pi(x)\tau\pi^0(x)]\psi^{r}(x) \qquad (2.29)$$
$$-eA_{\mu}^{em}\varepsilon_{ijk}\left[\pi_{\cdot i}(x)\partial_{\mu}\pi_j(x) - \frac{\pi_j(x)}{2F}\bar{\psi}^{r}(x)\gamma^{\mu}\gamma^{5}\tau_i\psi^{r}(x)\right].$$

The renormalized electromagnetic current is given by,

$$j_r^{\mu} = j_{\psi^r}^{\mu} + j_{\pi}^{\mu} + j_{\psi^r\pi}^{\mu} + \delta j_{\psi^r}^{\mu}. \qquad (2.30)$$

where, $j_{\psi^r}^{\mu}$ is the quark current, j_{π}^{μ} is the charged pion current, $j_{\psi^r\pi}^{\mu}$ is the quark-pion current and $\delta j_{\psi^r}^{\mu}$ is the contribution from the counterterm.

The partially conserved axial vector current A_i^{μ} is given by,

$$A_i^{\mu} = F\partial^{\mu}\pi_i + \bar{\psi}^{r}\gamma^{\mu}\gamma^{5}\frac{\tau_i}{2}\psi^{r} - \frac{\varepsilon_{ijk}}{2F}\bar{\psi}^{r}\gamma^{\mu}\tau_j\psi^{r}\tau_k$$
$$+\frac{1}{4F^2}\bar{\psi}^{r}\gamma^{\mu}\gamma^{5}(\pi\,\tau\,\pi_i - \pi^2\tau_i)\psi^{r} \qquad (2.31)$$
$$+\bar{\psi}^{r}(\hat{Z}-1)\gamma^{\mu}\gamma^{5}\frac{\tau_i}{2}\psi^{r} + o(\pi^2)$$

The axial form factor $G_A(Q^2)$ of the nucleon is given below,

$$\left\langle N_{\acute{s}}\left(\frac{\vec{q}}{2}\right)\middle|\int d^3\boldsymbol{x}\; e^{iqx}A_3(x)\middle|N_s\left(-\frac{\vec{q}}{2}\right)\right\rangle = \chi_{N_{\acute{s}}}^{\dagger}\sigma_N\frac{\tau^3{}_N}{2}\chi_{N_s}G_A(Q^2). \qquad (2.32)$$

They could predict the value of the axial charge, g_A = 1.19. Furthermore, they could prove that the contributions of excited quark states in the one-loop diagrams are essential, in order to adjust a small correction to the tree-level diagrams and lead to the search for an appropriate value of parameter ρ .

In 2014, X.Y. Liu et al., used the PCQM to study the electromagnetic properties of baryon octets (15). They have used a predetermined relativistic quark wave function, the Sturmian functions, instead of the typical Gaussian ansatz. The radial quark wave functions have two components. In the ground state, the wave functions were expanded in the set of Sturmian functions, $S_{nl}(r)$. The upper $g(r)$ and the lower $f(r)$ are defined as

$$g(r) = \sum_n A_n\frac{S_{n0}(r)}{r},$$
$$f(r) = f\sum_n B_n\frac{S_{n0}(r)}{r}, \qquad (2.33)$$

, with

$$S_{nl}(r) = \left[\frac{n!}{(n+2l+1)!}\right]^{\frac{1}{2}} (2br)^{l+1} e^{-br} L_n^{2l+1}(2br). \tag{2.34}$$

where, $L_n^{2l+1}(x)$ are the Laguerre polynomials.

They show that the first five Sturmian functions with n = 0, 1, 2, 3, 4 could give a good value of the proton charge form factor when compared to the experimental data. Furthermore, after obtaining the proton charge radius, they extended to predict the charge radii of other baryon octets. The theoretical calculations showed that p, Σ^- charge radii were reasonably agreed with the experimental data. Besides, for the charge radii of the charged baryons, they claimed that the 3q-core diagram dominantly contributes around 90% and less than 10% comes from the meson cloud effect. However, the theoretical results of the charge radii of neutral baryons ($n, \Sigma^0, \Lambda, \Xi^0$) were small.

In 2015, X. Y. Liu et al. used the PCQM to study the meson cloud contributions to the baryon axial form factors, with the quark wave functions, expanded in the set of Sturmian functions (16). In the study, they used the quark-meson interaction Lagrangian below,

$$\begin{aligned} \mathcal{L}_I^W(x) &= \frac{1}{2F} \partial_\mu \Phi_i(x) \bar{\psi}(x) \gamma^\mu \gamma^5 \lambda_i \psi(x) \\ &+ \frac{f_{ijk}}{4F^2} \Phi_i(x) \partial_\mu \Phi_j(x) \bar{\psi}(x) \gamma^\mu \lambda_k \psi(x). \end{aligned} \tag{2.35}$$

The axial-vector current A_i^μ can be determined by,

$$\begin{aligned} A_i^\mu &= F \partial^\mu \Phi_i + \bar{\psi} \gamma^\mu \gamma^5 \frac{\lambda_i}{2} \psi - \frac{f_{ijk}}{2F} \bar{\psi} \gamma^\mu \lambda_j \psi \Phi_k \\ &+ \bar{\psi}(\hat{Z} - 1) \gamma^\mu \gamma^5 \frac{\lambda_i}{2} \psi + o({\Phi_i}^2) \\ &+ \bar{\psi}^r (\hat{Z} - 1) \gamma^\mu \gamma^5 \frac{\tau_i}{2} \psi^r + o(\pi^2). \end{aligned} \tag{2.37}$$

The diagrams that contribute to the axial form factor are (i) 3q- core leading order (ii) 3q-core counterterm, (iii) self-energy I, (iv) self-energy II, (v) meson exchange and (vi) vertex correction.

The axial radii of octet baryons were defined by

$$\langle r_A^2 \rangle_B = -6 \frac{1}{g_A^B} \frac{dG_A^B(Q^2)}{dQ^2} \bigg|_{Q^2=0} \tag{2.38}$$

The nucleon axial radius $\langle r_A^2 \rangle_N$ was a little bit larger than the experimental data. Besides, they also predicted the axial radius of $\langle r_A^2 \rangle_\Sigma$ and $\langle r_A^2 \rangle_\Xi$ in the same order as of the nucleon. Furthermore, they studied the contribution from pion, kaon and eta meson to the axial charges separately. They found that the contribution from pion was the most significant.

CHAPTER 3
METHODOLOGY: THE PERTURBATIVE CHIRAL QUARK MODEL

In this study, we use the PCQM to investigate the nucleon properties and electromagnetic form factors. The main objectives are i) to search for suitable parameters $(\rho\,,\mathrm{R})$ of the Gaussian ansatz quark wave function when includes the excited states, ii) to study the static properties and the nucleon form factors and iii) to study the contributions of excited quark propagators to the electromagnetic properties. The methodology used in this work is presented in detail below.

1. Construction of an effective Lagrangian of the PCQM

The PCQM is based on an effective chiral Lagrangian. In this model, we consider the quarks within a nucleon as relativistic quarks, surrounded by a cloud of pseudoscalar mesons $(\pi\,,\mathrm{K},\eta)$ as required by spontaneous chiral symmetry breaking. The model Lagrangian of the PCQM is given by

$$\mathcal{L}_{eff} = \mathcal{L}_0 + \mathcal{L}_{int}^{str} + \mathcal{L}_{\chi SB}, \tag{3.1}$$

where

$$\mathcal{L}_0 = \bar{\psi}(x)\left[i\gamma^\mu\partial_\mu - \gamma^0 V(r) - S(r)\right]\psi(x) + \frac{1}{2}\sum_{i=1}^{8}\left[\partial_\mu \Phi_i(x)\right]^2, \tag{3.2}$$

is the Lagrangian for the massless current quark field $\psi(x)$ moving in the potential $S(r) + \gamma^0 V(r)$ and the massless mesons field $\Phi_i(x)$. Here, we defined $r = |\vec{x}|$. The interaction Lagrangian term $\mathcal{L}_{int}^{str}$ is the strong interaction Lagrangian between the valence quark and the meson field, which can be written as

$$\mathcal{L}_{int}^{str} = -\bar{\psi}(x)S(r)i\gamma^5\frac{\hat{\Phi}(x)}{F}\,\psi(x), \tag{3.3}$$

where $F = 88\,\mathrm{MeV}$ is the π-decay constant (1), and $\hat{\Phi}(x)$ is the pseudoscalar mesons in the matrix form, defined as below.

$$\frac{\hat{\Phi}}{\sqrt{2}} = \sum_{i=1}^{8}\frac{\Phi_i\lambda_i}{2} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{pmatrix}, \tag{3.4}$$

Finally, due to the masses of the current quarks which results in the explicit chiral symmetry breaking, we obtain the Lagrangian $\mathcal{L}_{\chi SB}$ for the mass terms as

$$\mathcal{L}_{\chi SB} = -\bar{\psi}(x)\mathcal{M}\psi(x) - \frac{B}{2}Tr\left[\hat{\Phi}^2(x)\mathcal{M}\right], \tag{3.5}$$

where $\mathcal{M} = \mathrm{diag}\{\hat{m}, \hat{m}, m_s\}$ represents the quark masses. Note that we considered in the isospin symmetry limit and define $\hat{m} \equiv m_u = m_d$, with the chosen values of $\hat{m} = 7\ \mathrm{MeV}$ and $m_s = 25\hat{m}$. In addition, $B = 1.4\ GeV$ is the quark condensate parameter (1). The meson masses satisfied the following relations

$$M_\pi^2 = 2\hat{m}B, \quad M_K^2 = (\hat{m} + m_s)B, \quad M_\eta^2 = \tfrac{2}{3}(\hat{m} + 2m_s)B, \tag{3.6}$$

The renormalization of the PCQM was done by using the counter-term technique. See Ref. (1) for a detailed procedure for renormalizing the PCQM.

In the calculation, we use the variational Gaussian ansatz in the ground state which is given by

$$u_0(\vec{x}) = N_0\, exp\left(-\frac{\vec{x}^2}{2R^2}\right)\begin{pmatrix} 1 \\ i\rho\dfrac{\vec{\sigma}\cdot\vec{x}}{R} \end{pmatrix}\chi_s\chi_f\chi_c, \tag{3.7}$$

The normalization condition $\int d^3x u_0^\dagger(\vec{x})u_0(\vec{x}) = 1$ is used for fixing the normalization constant, $N_0 = \left[\pi^{3/2}R^3(1 + 3\rho^2/2)\right]^{-1/2}$. Other parts of the wave function are the spin part χ_s, flavor part χ_f, and color part χ_c. In this model, ρ and R are the free-parameters. The quark wave function $u_0(\vec{x})$ is the solution of the Dirac equation

$$\left[-i\vec{\alpha}\cdot\vec{\nabla} + \beta S(r) + V(r) - \mathcal{E}_0\right]u_0(\vec{x}) = 0. \tag{3.8}$$

the space-time quark wave function can be written in the form of,

$$\psi_0(\vec{x}, t) = u_0(\vec{x})e^{-i\mathcal{E}_0 t}, \tag{3.9}$$

where $\mathcal{E}_0$ refers to the ground state quark energy. By inserting $u_0(\vec{x})$ into the Dirac equation, we have the explicit form of the potential $S(r)$ and $V(r)$ for the Gaussian ansatz and both of them are in the forms of

$$S(r) = \frac{1 - 3\rho^2}{2\rho R} + \frac{\rho}{2R^3}r^2 = M_1 + c_1 r^2, \tag{3.10}$$

$$V(r) = \mathcal{E}_0 - \frac{1 + 3\rho^2}{2\rho R} + \frac{\rho}{2R^3}r^2 = M_2 + c_2 r^2. \tag{3.11}$$

To fix the parameters ρ and R of the PCQM, the parameter ρ links to the axial coupling constant g_A by

$$g_A = \frac{5}{3}\left(1 - \frac{2\rho^2}{1 + \frac{3}{2}\rho^2}\right). \tag{3.12}$$

With the value of g_A approximated to be 1.25 and restricted to the zero-order diagram, the value of ρ becomes $\rho = \sqrt{2/13} \approx 0.39$. Together with the proton charge radius in the leading-order (LO) term, we obtain

$$\langle r_E^2 \rangle_{LO} = \frac{3R^2}{2}\left(\frac{1 + \frac{5}{2}\rho^2}{1 + \frac{3}{2}\rho^2}\right). \tag{3.13}$$

By using the value of $\rho = \sqrt{2/13}$ and assume that the values of $\langle r_E^2 \rangle_{LO}$ lies between $0.5\ \mathrm{fm}^2$ and $0.7\ \mathrm{fm}^2$, the values of R is found to be $0.55\ \mathrm{fm}$ to $0.65\ \mathrm{fm}$. We, therefore, took set $\rho = \sqrt{2/13}$ and $R = 0.60\ \mathrm{fm}$ as the fixed model parameters. In our study, both values of ρ, R needed to be reconsidered when we modify the quark propagator by taking the excited states into account.

Starting from the specific forms of the potentials $S(r)$ and $V(r)$, the quark wave function in the excited states α, $u_\alpha(\vec{x})$ can be obtained by solving the corresponding Dirac equation. The general form of the quark in the α state is

$$u_\alpha(\vec{x}) = N_\alpha\, exp\left(-\frac{\vec{x}^2}{2R^2}\right)\begin{pmatrix} g_\alpha(r) \\ i\vec{\sigma}\cdot\hat{x} f_\alpha(r)\end{pmatrix}\mathcal{Y}_\alpha(\hat{x})\chi_f\chi_c. \tag{3.14}$$

with the energy ε_α. Therefore, the quark field of in the excited states α becomes

$$\psi_\alpha(\vec{x},t) = u_\alpha(\vec{x})e^{-i\mathcal{E}_\alpha t} \quad , \quad \bar{\psi}_\alpha(\vec{x},t) = \bar{u}_\alpha(\vec{x})e^{i\mathcal{E}_\alpha t}. \tag{3.15}$$

The normalization constant N_α can be fixed by using the condition $\int d^3x u_\alpha^\dagger(\vec{x})u_\alpha(\vec{x}) = 1$, as

$$N_\alpha = \left[2^{-2\left(n+l+\frac{1}{2}\right)}\pi^{\frac{1}{2}}R_\alpha^3\frac{(2n+2l)!}{(n+l)!\,(n-1)!}\left\{1 + \rho_\alpha^2(2n + l - \frac{1}{2})\right\}\right]^{-1/2}. \tag{3.16}$$

The number n is the principal quantum number of states, with $n = 1,2, \ldots$ and l is the angular momentum. Here, we define $\Delta\mathcal{E}_\alpha = \mathcal{E}_\alpha - \mathcal{E}_0$ to be the difference between the energy of excited state α, denoted by$\mathcal{E}_\alpha$ and the ground state energy, denoted by $\mathcal{E}_0$.

The $\Delta\mathcal{E}_\alpha$ are related to ρ and R by the relation

$$\left(\Delta\mathcal{E}_\alpha + \frac{3\rho}{R}\right)^2 \left(\Delta\mathcal{E}_\alpha + \frac{1}{\rho R}\right) = \frac{\rho}{R^3}(4n + 2l - 1)^2. \quad (3.17)$$

We summarize the ground state and excited states of quark propagators in Table 1.

TABLE 1 The corresponding energies of the quark in the low-lying states of the PCQM

Quark state	Quantum state	Label	$\Delta\mathcal{E}_\alpha(GeV)$
Ground state	$n = 1, l = 0, s = +\frac{1}{2}$	$1s_{1/2}$	0.00
1st excited state	$n = 1, l = 1, s = -\frac{1}{2}$	$1p_{1/2}$	0.227204
	$n = 1, l = 1, s = +\frac{1}{2}$	$1p_{3/2}$	
2nd excited state	$n = 1, l = 1, s = -\frac{1}{2}$	$1d_{3/2}$	0.425221
	$n = 1, l = 1, s = +\frac{1}{2}$	$1d_{5/2}$	
	$n = 2, l = 0, s = +\frac{1}{2}$	$2s_{1/2}$	

The angular dependent, resulting from the coupling of the spin and the orbital parts, of the wave function is $\mathcal{Y}_\alpha(\hat{x})$. The component, $g_\alpha(r)$ of the quark wave function is given by

$$g_\alpha(r) = \left(\frac{r}{R_\alpha}\right)^l L_{n-1}^{l+1/2}\left(\frac{r^2}{R_\alpha^2}\right) e^{-\frac{r^2}{2R_\alpha^2}}. \quad (3.18)$$

whereas the lower components, $f_\alpha(r)$, which depend on the value of the total angular momentum j, has the following form,

(i) in case of the total angular momentum $j = l + \frac{1}{2}$,

$$f_\alpha(r) = \rho_\alpha \left(\frac{r}{R_\alpha}\right)^{l+1} \left[L_{n-1}^{l+3/2}\left(\frac{r^2}{R_\alpha^2}\right) + L_{n-2}^{l+3/2}\left(\frac{r^2}{R_\alpha^2}\right)\right] e^{-\frac{r^2}{2R_\alpha^2}}, \quad (3.19)$$

(ii) and $j = l - \frac{1}{2}$,

$$f_\alpha(r) = -\rho_\alpha \left(\frac{r}{R_\alpha}\right)^{l-1} \left[(n + l - \tfrac{1}{2}) L_{n-1}^{l-1/2}\left(\frac{r^2}{R_\alpha^2}\right) + n L_n^{l-1/2}\left(\frac{r^2}{R_\alpha^2}\right)\right] e^{-\frac{r^2}{2R_\alpha^2}}, \tag{3.20}$$

We summarize the $f_\alpha(r)$ and $g_\alpha(r)$ of each state in the Table 2.

TABLE 2 The lower and the upper components of the quark wave function

State α (u_α)	Label	$g_\alpha(r)$	$f_\alpha(r)$
u_0	$1s_{1/2}$ $n = 1,$ $l = 0,$ $s = +1/2$	$g_0(r) = e^{-\frac{r^2}{2R^2}}$	$f_0(r) = \dfrac{\rho r}{R} e^{-\frac{r^2}{2R^2}}$
u_1	$1p_{1/2}$ $n = 1,$ $l = 1,$ $s = -1/2$	$g_1(r) = \dfrac{r}{R_1} e^{-\frac{r^2}{2{R_1}^2}}$	$f_1(r)$ $= -\rho_1 \left(3 - \left(\dfrac{r}{R_1}\right)^2\right) e^{-\frac{r^2}{2{R_1}^2}}$

TABLE 2 (Continued)

State α (u_α)	Label	$g_\alpha(r)$	$f_\alpha(r)$
u_2	$1p_{3/2}$ $n=1,$ $l=1,$ $s=+1/2$	$g_2(r)=\frac{r}{R_2}e^{-\frac{r^2}{2{R_2}^2}}$	$f_2(r)=\rho_2\left(\frac{r}{R_2}\right)^2 e^{-\frac{r^2}{2{R_2}^2}}$
u_3	$1d_{3/2}$ $n=1,$ $l=2,$ $s=-1/2$	$g_3(r)=\left(\frac{r}{R_3}\right)^2 e^{-\frac{r^2}{2{R_3}^2}}$	$f_3(r)=\frac{-\rho_3 r}{R_3}\left(5-\left(\frac{r}{R_3}\right)^2\right)e^{-\frac{r^2}{2{R_3}^2}}$
u_4	$1d_{5/2}$ $n=1,$ $l=2$ $s=+1/2$	$g_4(r)=\left(\frac{r}{R_4}\right)^2 e^{-\frac{r^2}{2{R_4}^2}}$	$f_4(r)=\rho_4\left(\frac{r}{R_4}\right)^3 e^{-\frac{r^2}{2{R_4}^2}}$
u_5	$2s_{1/2}$ $n=2,$ $l=0,$ $s=+1/2$	$g_5(r)$ $=\left(\frac{3}{2}-\left(\frac{r}{R_5}\right)^2\right)e^{-\frac{r^2}{2{R_5}^2}}$	$f_5(r)=\frac{\rho_5 r}{R_5}\left(\frac{7}{2}-\left(\frac{r}{R_5}\right)^2\right)e^{-\frac{r^2}{2R_5^2}}$

In this study, we also investigate the effects of higher excited states on the magnetic moments of the nucleon. As we know that the higher excited states also contribute to the value of the magnetic moments of the nucleon but still be unclear how much they could contribute. For higher excited states up to the 5th excited states and the notation are shown in Table 4 and the upper and lower spinors are shown in Appendix F.

The parameters ρ and R for a quark in the ground state are related to the parameters ρ_α and R_α for the quark in the state α by

$$\rho_\alpha = \rho\left(\frac{R_\alpha}{R}\right)^3, \tag{3.21}$$

$$R_\alpha = R(1 + \Delta\mathcal{E}_\alpha \rho R)^{-1/4}\,. \tag{3.22}$$

In this model, the free meson propagator as indicated by the Quantum Field Theory is

$$i\Delta_{ij}(x-y) = \delta_{ij}\int \frac{d^4k}{(2\pi)^4 i}\frac{exp[-ik(x-y)]}{M_\Phi^2 - k^2 - i\epsilon}, \tag{3.23}$$

where M_Φ denotes the meson mass. ($\Phi = \pi, K, \eta$). In the case of the quark, we apply the bound states quark propagator, which is written in terms of the quark field. For simplicity, the ground state of the quark was assumed to dominate the quark propagator; therefore, only the quark propagator in the ground state is used i.e.

$$iG_\psi(x,y) = u_0(\vec{x})\bar{u}_0(\vec{y})\,exp[-i\mathcal{E}_0(x_0 - y_0)]\,\theta(x_0 - y_0). \tag{3.24}$$

A straightforward modification to the quark propagator by summing up all the excited states of the quark results in

$$iG_\psi(x,y) = \theta(x_0 - y_0)\sum_\alpha u_\alpha(\vec{x})\bar{u}_\alpha(\vec{y})\,exp[-i\mathcal{E}_\alpha(x_0 - y_0)]. \tag{3.25}$$

To test the validity of the model, the first two excited states ($1p_{1/2}, 1p_{3/2}, 1d_{3/2}, 1d_{5/2}$ and $2s_{1/2}$ states), together with the $1s_{1/2}$ state have been included in the calculation of the octet baryons electromagnetic form factors, as mentioned in (2). Nevertheless, such modification of the quark propagator in the calculational technique was shown the significance of the modification to the charge form factor of the neutron only.

Obviously, for consistency on the nucleonic level, such modification must be done to the study of the electromagnetic form factors of both the proton and the neutron. We expect that the previous values of the parameters of the model must be changed.

We intend to fix the parameters ρ and R by using the nucleon magnetic moments reported from the experiments. For simplicity, however, we set the value of R to be $0.60\ \mathrm{fm}$ as same as in the previous works and attempt to find the value of the parameter ρ that reproduces the nucleon magnetic moments.

2. The Electromagnetic Form Factors of the Nucleon

The standard minimal substitution procedure is used for introduction of the electromagnetic interaction by the modifications of

$$\partial_\mu \psi^r \to D_\mu \psi^r = \partial_\mu \psi^r + ieQA_\mu \psi^r, \tag{3.26}$$

$$\partial_\mu \Phi_i \to D_\mu \Phi_i = \partial_\mu \Phi_i + e\left(f_{3ij} + \frac{f_{8ij}}{\sqrt{3}}\right) A_\mu \Phi_j \tag{3.27}$$

Here, A_μ is the field of the photon and ψ^r is the renormalized quark field. The matrix, Q, indicates the quark's charges as $Q = \mathrm{diag}\{2/3, -1/3, -1/3\}$ and f_{ijk} are the structure constants in the SU(3) formalism. Note that

$$j_r^\mu = j_{\psi^r}^\mu + j_\Phi^\mu + \delta j_{\psi^r}^\mu, \tag{3.28}$$

represents the electromagnetic current, where $j_{\psi^r}^\mu$ is the current of the quark, j_Φ^μ is the current of the (charge) meson and $\delta j_{\psi^r}^\mu$ is the current due to the counter-term, see [9] for detailed discussion. Theses currents can be written explicitly as

$$j_{\psi^r}^\mu = \bar{\psi}^r \gamma^\mu Q \psi^r, \tag{3.29}$$

$$j_\Phi^\mu = \left(f_{3ij} + \frac{f_{8ij}}{\sqrt{3}}\right) \Phi_i \partial^\mu \Phi_j, \tag{3.30}$$

and

$$\delta j_{\psi^r}^\mu = \bar{\psi}^r (Z-1) \gamma^\mu Q \psi^r, \tag{3.31}$$

where Z is the renormalization constant matrix, $Z = \mathrm{diag}\{\hat{Z}, \hat{Z}, Z_s\}$, and with the isospin symmetry, $\hat{Z} = Z_u = Z_d$. We obtain such constants by considering the charge renormalization of the corresponding baryons. Only $\hat{Z}$ is relevant in the nucleonic sector, since there is no $\boldsymbol{S}$-quark in the nucleon.

The nucleon electromagnetic form factors are calculated within the Breit frame and the so-called Sachs form factors, $G_E^N(Q^2)$ and $G_M^N(Q^2)$, of the nucleon are defined by

$$\chi_{N_{s'}}^{\dagger}\chi_{N_s}G_E^N(Q^2) = \left\langle N_{s'}\left(\frac{\vec{q}}{2}\right)\middle|J^0(0)\middle|N_s\left(-\frac{\vec{q}}{2}\right)\right\rangle, \tag{3.32}$$

and

$$\chi_{N_{s'}}^{\dagger}\frac{i\vec{\sigma}_N\times\vec{q}}{2M_N}\chi_{N_s}G_M^N(Q^2) = \left\langle N_{s'}\left(\frac{\vec{q}}{2}\right)\middle|\vec{J}(0)\middle|N_s\left(-\frac{\vec{q}}{2}\right)\right\rangle, \tag{3.33}$$

where $G_E^N(Q^2)$ and $G_M^N(Q^2)$ denote the charge and the magnetic Sachs form factors, respectively. Here, on the left-hand side, the calculation must be performed on the nucleonic level. For the elastic scattering and in the Breit frame, $Q^2 = -\vec{q}^2 > 0$. The charge Sachs form factors of the nucleon are normalized at the zero recoil $(Q^2 = 0)$ as

$$G_E^p(0) = 1, \qquad G_E^n(0) = 0. \tag{3.34}$$

are the charge of the proton and the neutron, respectively. On the other hand, we have

$$G_M^p(0) = \mu_p = 2.793, \qquad G_M^n(0) = \mu_n = -1.913, \tag{3.35}$$

where μ_p and μ_n represent proton and neutron magnetic moments, respectively.

In the PCQM formalism, the Sachs form factors can be calculated from the relations

$$\chi_{N_{s'}}^{\dagger}\chi_{N_s}G_E^N(Q^2)$$
$$= \left\langle \phi_0\middle|\sum_{n=0}^{2}\frac{i^n}{n!}\int\delta(t)d^4xd^4x_1\ldots d^4x_ne^{-iq\cdot x}T[\mathcal{L}_r^{str}(x_1)\ldots\mathcal{L}_r^{str}(x_n)j_r^0(x)]\middle|\phi_0\right\rangle_c^N \tag{3.36}$$

and

$$\chi_{N_{s'}}^{\dagger}\frac{i\vec{\sigma}_N\times\vec{q}}{2M_N}\chi_{N_s}G_M^N(Q^2)$$
$$= \left\langle \phi_0\middle|\sum_{n=0}^{2}\frac{i^n}{n!}\int\delta(t)d^4xd^4x_1\ldots d^4x_ne^{-iq\cdot x}T[\mathcal{L}_r^{str}(x_1)\ldots\mathcal{L}_r^{str}(x_n)\vec{j}_r(x)]\middle|\phi_0\right\rangle_c^N \tag{3.37}$$

Besides, the renormalized strong interaction Lagrangian $\mathcal{L}_r^{str}(x)$ in the PCQM is in the form

$$\mathcal{L}_r^{str}(x) = \mathcal{L}_I^{str}(x) + \delta\mathcal{L}^{str}, \tag{3.38}$$

where

$$\mathcal{L}_I^{str}(x) = -\bar{\psi}^r(x)S(r)i\gamma^5\frac{\hat{\Phi}(x)}{F}\psi^r(x), \tag{3.39}$$

and $\delta\mathcal{L}^{str}$ is the strong interaction Lagrangian corresponding to the counter-terms, see Ref. (1) for details.

The charge radii and magnetic radii of nucleons are given by,

$$\langle r_{E,M}^2\rangle^N = \frac{-6}{G_{E,M}^N(0)}\frac{dG_{E,M}^N(0)}{dQ^2}\bigg|_{Q^2=0}, \tag{3.40}$$

Since the charge of the neutron is zero, the charge radius of the neutron is defined by,

$$\langle r_E^2\rangle^n = -6\frac{dG_E^n(0)}{dQ^2}\bigg|_{Q^2=0}. \tag{3.41}$$

The relevant Feynman diagrams for the nucleon Sachs form factors are presented in Fig. 1.

The α and β in the diagrams shown in Fig.1 indicate the insertions of the quark in the α and β states to the quark propagator. The diagrams that involve in the dressing of the quark propagator are the meson-cloud diagram and the vertex-correction diagram. We present the analytical results in detail as follows.

1. The three-quark (3q) core diagram:

The analytical expressions for the leading-order (LO) terms and the next-to-leading (NLO) terms of the Sachs form factors for the 3q core diagram are indicated by

$$G_{E,M}^N(Q^2) = G_{E,M}^N(Q^2)\Big|_{3q}^{LO} + G_{E,M}^N(Q^2)\Big|_{3q}^{NLO}, \tag{3.42}$$

where, for proton ($N = p$)

$$G_E^p(Q^2)\Big|_{3q}^{LO} = exp\left(-\frac{Q^2R^2}{4}\right)\left(1-\frac{Q^2R^2\rho^2}{4\left(1+\frac{3}{2}\rho^2\right)}\right), \tag{3.43}$$

$$G_E^p(Q^2)\Big|_{3q}^{NLO} = exp\left(-\frac{Q^2R^2}{4}\right)\hat{m}^r\frac{Q^2R^3\rho}{4\left(1+\frac{3}{2}\rho^2\right)^2} \tag{3.44}$$

$$\times\left(\frac{1+7\rho^2+\frac{15}{4}\rho^4}{1+\frac{3}{2}\rho^2}-\frac{Q^2R^2\rho^2}{4}\right),$$

$$G_M^p(Q^2)\big|_{3q}^{LO}=exp\left(-\frac{Q^2R^2}{4}\right)\frac{2m_N\rho R}{1+\frac{3}{2}\rho^2}, \tag{3.45}$$

$$G_M^p(Q^2)\big|_{3q}^{NLO}=G_M^p(Q^2)\big|_{3q}^{LO}\frac{\widehat{m}^r\rho R}{1+\frac{3}{2}\rho^2}\left(\frac{Q^2R^2}{4}-\frac{2-\frac{3}{2}\rho^2}{1+\frac{3}{2}\rho^2}\right) \tag{3.46}$$

and for neutron ($N=n$)

$$G_E^n(Q^2)|_{3q}^{LO}=G_E^n(Q^2)|_{3q}^{NLO}=0, \tag{3.47}$$

$$G_M^n(Q^2)|_{3q}^{LO}=-\frac{2}{3}G_M^p(Q^2)\big|_{3q}^{LO}, \tag{3.48}$$

$$G_M^n(Q^2)|_{3q}^{NLO}=-\frac{2}{3}G_M^p(Q^2)\big|_{3q}^{NLO}. \tag{3.49}$$

Besides, the quark mass renormalization is

$$\widehat{m}^r=\widehat{m}-\frac{1}{(2\pi)^2F^2}\sum_\alpha\int_0^\infty dpp^2F_\alpha^\dagger(p^2)\,F_\alpha(p^2)$$
$$\times\left\{\begin{array}{l}\frac{9}{\omega_\pi(p^2)[\omega_\pi(p^2)+\Delta\mathcal{E}_\alpha]}\\+\frac{6}{\omega_K(p^2)[\omega_K(p^2)+\Delta\mathcal{E}_\alpha]}\\+\frac{1}{\omega_\eta(p^2)[\omega_\eta(p^2)+\Delta\mathcal{E}_\alpha]}\end{array}\right\}, \tag{3.50}$$

where

$$\omega_\Phi(p^2)=\sqrt{\vec{p}^2+M_\Phi^2}\ \ ,\ x=cos\,\theta\ and\ \Delta\mathcal{E}_\alpha\equiv\mathcal{E}_\alpha-\mathcal{E}_0. \tag{3.51}$$

and

$$F_\alpha(p^2)=-N_0N_\alpha\int_0^\infty dr\,r^2S(r)[f_0(r)g_\alpha(r)+f_\alpha(r)g_0(r)]$$
$$\times\int_0^\pi sin\theta cos\theta d\theta\int_0^{2\pi}d\phi\,(C_0Y_{0,0})(A_1Y_{\alpha,0})e^{-iprcos\theta} \tag{3.52}$$

Note that the subscript $\alpha=0$ refers to quark in the ground state $(1s_{1/2})$. The corresponding Clebsch-Gordan coefficients, C_0 and A_1, defined by $C_0=\left(00\frac{1}{2}\frac{1}{2}\middle|\frac{1}{2}\frac{1}{2}\right)$ and $A_1=\left(l_\alpha 0\frac{1}{2}\frac{1}{2}\middle|j_\alpha\frac{1}{2}\right)$ respectively, where $\left(l_\alpha\,m_{l_\alpha}\,s\,m_s\,\middle|j_\alpha m_{j_\alpha}\right)$ represents explicitly the coupling that leads to the Clebsch-Gordan coefficients. The Y_{l_α,m_α} are the spherical harmonics function and m_N is the mass of the nucleon. In this work, the nucleon mass has the value of $m_N = 0.938$ GeV.

2. The three-quark counter-term (CT) diagram:

$$G_{E,M}^{p}(Q^2)\big|_{CT} = (\hat{Z}-1)G_{E,M}^{p}(Q^2)\big|_{3q}^{LO}, \tag{3.53}$$

$$G_E^n(Q^2)|_{CT} = 0\,, \tag{3.54}$$

$$G_M^n(Q^2)|_{CT} = (\hat{Z}-1)G_M^n(Q^2)|_{3q}^{LO}. \tag{3.55}$$

The renormalization constant $\hat{Z}$ is

$$\begin{aligned}\hat{Z} = 1 - \frac{1}{(2\pi)^2F^2}\sum_{\alpha}\int_0^{\infty} dpp^2F_\alpha^{\dagger}(p^2)\,F_\alpha(p^2)\\ \times\left\{\frac{1}{\omega_\pi(p^2)[\omega_\pi(p^2)+\Delta\mathcal{E}_\alpha]^2} - \frac{2}{\omega_K(p^2)[\omega_K(p^2)+\Delta\mathcal{E}_\alpha]^2}\right.\\ \left. + \frac{1/3}{\omega_\eta(p^2)\left[\omega_\eta(p^2)+\Delta\mathcal{E}_\alpha\right]^2}\right\}\end{aligned} \tag{3.56}$$

3. The meson-cloud (MC) diagram:

$$\begin{aligned}&G_E^N(Q^2)|_{\alpha,MC}\\ &= \left\langle\phi_0\middle|\sum_{n=0}^{2}\frac{i^n}{n!}\int\delta(t)d^4xd^4x_1\ldots d^4x_ne^{-iq\cdot x}T[\mathcal{L}_r^{str}(x_1)\ldots\mathcal{L}_r^{str}(x_n)j_\Phi^0(x)]\middle|\phi_0\right\rangle_c^N\\ &\qquad 4\langle\phi_0| -\tfrac{1}{2}\int\delta(t)d^4xd^4x_1d^4x_2e^{-iq\cdot x}:\left(-\bar{\psi}(x_1)i\gamma^5\tfrac{\lambda_k}{F}\Phi_k(x_1)S(r)\psi_\alpha(x_1)\right)\\ &\times\left(-\bar{\psi}_\alpha(x_2)i\gamma^5\frac{\lambda_l}{F}\Phi_l(x_2)S(r)\psi(x_2)\right)\times\left(f_{3ij}+\frac{f_{8ij}}{\sqrt{3}}\right)\Phi_i(x)\frac{\partial\Phi_j(x)}{\partial t}:|\phi_0\rangle\end{aligned} \tag{3.57}$$

For proton $N = p$;

$$\begin{aligned}&G_E^p(Q^2)\big|_{\alpha,MC}\\ &= \frac{1}{(2\pi)^2F^2}\int_0^{\infty}dpp^2\int_{-1}^{1}dx\frac{p+\sqrt{Q^2}x}{\sqrt{p^2+Q^2+2p\sqrt{Q^2}x}} * F1 * F2\\ &\times\left[\frac{2}{(\omega_\pi(\vec{p})+\Delta\varepsilon_\alpha)(\omega_\pi(\vec{p}+\vec{q})+\Delta\varepsilon_\alpha)(\omega_\pi(\vec{p})+(\omega_\pi(\vec{p}+\vec{q}))}\right.\\ &\left.+\frac{4}{(\omega_K(\vec{p})+\Delta\varepsilon_\alpha)(\omega_K(\vec{p}+\vec{q})+\Delta\varepsilon_\alpha)(\omega_K(\vec{p})+(\omega_K(\vec{p}+\vec{q}))}\right]\end{aligned} \tag{3.58}$$

Where $F1$ and $F2$ are defined as below,

$$\begin{aligned} F1 \equiv (-N_0 N_\alpha) \int_0^\infty r^2 dr \int_0^\pi sin\theta cos\theta d\theta \\ \times \int_0^{2\pi} d\phi\, S(r)\big(f_0(r)g_\alpha(r) + f_\alpha(r)g_0(r)\big) \\ \times (C_0 Y_0^0)(A_1 Y_\alpha^0) e^{i|\vec{p}+\vec{q}|rcos\theta} \end{aligned} \tag{3.59}$$

$$\begin{aligned} F2 \equiv (-N_0 N_\alpha) \int_0^\infty r^2 dr \int_0^\pi sin\theta cos\theta d\theta \\ \times \int_0^{2\pi} d\phi\, S(r)\big(f_0(r)g_\alpha(r) + f_\alpha(r)g_0(r)\big) \\ \times (C_0 Y_0^0)(A_1 Y_\alpha^0) e^{-iprcos\theta} \end{aligned} \tag{3.60}$$

For neutron $N = n$;

$$\begin{aligned} G_E^n(Q^2)|_{\alpha,MC} &= \frac{1}{(2\pi)^2 F^2} \int_0^\infty dpp^2 \int_{-1}^1 dx \frac{p + \sqrt{Q^2}x}{\sqrt{p^2 + Q^2 + 2p\sqrt{Q^2}x}} * F1 \\ &\times F2 \left[\frac{-2}{(\omega_\pi(\vec{p})+\Delta\varepsilon_\alpha)(\omega_\pi(\vec{p}+\vec{q})+\Delta\varepsilon_\alpha)(\omega_\pi(\vec{p})+(\omega_\pi(\vec{p}+\vec{q}))} + \frac{2}{(\omega_K(\vec{p})+\Delta\varepsilon_\alpha)(\omega_K(\vec{p}+\vec{q})+\Delta\varepsilon_\alpha)(\omega_K(\vec{p})+(\omega_K(\vec{p}+\vec{q}))} \right] \end{aligned} \tag{3.61}$$

By definition, $G_M^N(Q^2)|_{\alpha,MC}$ is given by,

$$\begin{aligned} &\chi_{N_{s'}}^\dagger \frac{i\vec{\sigma}_N \times \vec{q}}{2M_N} \chi_{N_s} G_M^N(Q^2)|_{\alpha,MC} \\ &= \left\langle \phi_0 \middle| \textstyle\sum_{n=0}^2 \frac{i^n}{n!} \int \delta(t) d^4x d^4x_1 d^4x_2 e^{-iq\cdot x} T[\mathcal{L}_r^{str}(x_1)\mathcal{L}_r^{str}(x_2) \vec{J}_\Phi(x)] \middle| \phi_0 \right\rangle_c^N \\ &= 4\langle \phi_0 | -\frac{1}{2} \int \delta(t) d^4x d^4x_1 d^4x_2 e^{-iq\cdot x} : \left(-\bar{\psi}(x_1) i\gamma^5 \frac{\lambda_k}{F} \Phi_k(x_1) S(r) \psi_\alpha(x_1)\right) \\ &\times \left(-\bar{\psi}_\alpha(x_2) i\gamma^5 \frac{\lambda_l}{F} \Phi_l(x_2) S(r) \psi(x_2)\right) \times \left(f_{3ij} + \frac{f_{8ij}}{\sqrt{3}}\right) \Phi_i(x) \nabla \Phi_j(x) : |\phi_0\rangle \\ &= \frac{-2}{(2\pi)^4 F^2} \langle \phi_0 \big| b_0^\dagger \int d^3p\, \vec{p} \times \frac{(\vec{\sigma}\cdot(\vec{p}+\vec{q}))}{|\vec{p}+\vec{q}|} \cdot \frac{(\vec{\sigma}\cdot\vec{p})}{|\vec{p}|} \\ &\times \int d^3x_1 \left(\bar{u}_0(\vec{x}_1) i\gamma^5 S(x_1) u_\alpha(\vec{x}_1) e^{i(\vec{p}+\vec{q})\cdot\vec{x}_1}\right) \int d^3x_2\, \bar{u}_\alpha(\vec{x}_2) i\gamma^5 S(x_2) u_0(\vec{x}_2) e^{-i\vec{p}\cdot\vec{x}_2} \\ &\times \left(f_{3ij} + \frac{f_{8ij}}{\sqrt{3}}\right) \frac{\lambda_i \lambda_j}{\omega_\phi^2(\vec{p}+\vec{q})\omega_\phi^2(\vec{p})} b_0 |\phi_0\rangle \end{aligned} \tag{3.62}$$

In order to calculate the matrix elements, we can restrict ourselves to the nucleon spin up only. Finally, we can get the $G_M^N(Q^2)|_{\alpha,MC}$ as below,

For proton ($N=p$);

$$G_M^p(Q^2)\big|_{MC} = \frac{2m_N}{(2\pi)^2F^2}\int_0^\infty dpp^3\int_{-1}^1 dx\frac{1-x^2}{\sqrt{p^2+Q^2+2p\sqrt{Q^2}x}}(-N_0N_\alpha)\int_0^\infty r^2dr$$

$$\times\int_0^\pi sin\theta cos\theta d\theta\int_0^{2\pi}d\phi\left(f_0(r)g_\alpha(r)+f_\alpha(r)g_0(r)\right)S(r)(C_0Y_0^0)(A_1Y_\alpha^0)e^{i|\vec{p}+\vec{q}|rcos\theta}$$

$$\times(-N_0N_\alpha)\int_0^\infty r^2dr\int_0^\pi sin\theta cos\theta d\theta\int_0^{2\pi}d\phi\, S(r)\left(f_0(r)g_\alpha(r)+f_\alpha(r)g_0(r)\right)$$

$$\times(C_0Y_0^0)(A_1Y_\alpha^0)e^{-iprcos\theta}$$

$$\times\left(\frac{\left(\frac{5}{3}\right)(\omega_\pi(\vec{p}+\vec{q})+\omega_\pi(\vec{p})+\Delta\varepsilon_\alpha)}{(\omega_\pi(\vec{p})+\Delta\varepsilon_\alpha)\omega_\pi(\vec{p})(\omega_\pi(\vec{p}+\vec{q})+\Delta\varepsilon_\alpha)\omega_\pi(\vec{p}+\vec{q})(\omega_\pi(\vec{p})+(\omega_\pi(\vec{p}+\vec{q}))}\right.$$

$$\left.+\frac{\left(\frac{4}{3}\right)(\omega_K(\vec{p}+\vec{q})+\omega_K(\vec{p})+\Delta\varepsilon_\alpha)}{(\omega_K(\vec{p})+\Delta\varepsilon_\alpha)\omega_K(\vec{p})(\omega_K(\vec{p}+\vec{q})+\Delta\varepsilon_\alpha)\omega_K(\vec{p}+\vec{q})(\omega_K(\vec{p})+(\omega_K(\vec{p}+\vec{q}))}\right)$$

(3.63)

For neutron ($N=n$) ;

$$G_M^n(Q^2)|_{MC}$$

$$=\frac{2m_N}{(2\pi)^2F^2}\int_0^\infty dpp^3\int_{-1}^1 dx\frac{1-x^2}{\sqrt{p^2+Q^2+2p\sqrt{Q^2}x}}(-N_0N_\alpha)\int_0^\infty r^2dr$$

$$\times\int_0^\pi sin\theta cos\theta d\theta\int_0^{2\pi}d\phi\left(f_0(r)g_\alpha(r)+(f_\alpha(r)g_0(r)\right)S(r)(C_0Y_0^0)(A_1Y_\alpha^0)e^{i|\vec{p}+\vec{q}|rcos\theta}$$

$$\times(-N_0N_\alpha)\int_0^\infty r^2dr\int_0^\pi sin\theta cos\theta d\theta\int_0^{2\pi}d\phi\, S(r)\left(f_0(r)g_\alpha(r)+f_\alpha(r)g_0(r)\right)$$

$$\times(C_0Y_0^0)(A_1Y_\alpha^0)e^{-iprcos\theta}$$

$$\times\left(\frac{-\left(\frac{5}{3}\right)(\omega_\pi(\vec{p}+\vec{q})+\omega_\pi(\vec{p})+\Delta\varepsilon_\alpha)}{(\omega_\pi(\vec{p})+\Delta\varepsilon_\alpha)\omega_\pi(\vec{p})(\omega_\pi(\vec{p}+\vec{q})+\Delta\varepsilon_\alpha)\omega_\pi(\vec{p}+\vec{q})(\omega_\pi(\vec{p})+(\omega_\pi(\vec{p}+\vec{q}))}\right.$$

$$\left.+\frac{-\left(\frac{1}{3}\right)(\omega_K(\vec{p}+\vec{q})+\omega_K(\vec{p})+\Delta\varepsilon_\alpha)}{(\omega_K(\vec{p})+\Delta\varepsilon_\alpha)\omega_K(\vec{p})(\omega_K(\vec{p}+\vec{q})+\Delta\varepsilon_\alpha)\omega_K(\vec{p}+\vec{q})(\omega_K(\vec{p})+(\omega_K(\vec{p}+\vec{q}))}\right).$$

(3.64)

4. The vertex-correction (VC) diagram:

From the definition of $G_E^N(Q^2)$

$$\chi^{\dagger}_{N_{s'}}\chi_{N_s}G_E^N(Q^2)|_{\alpha\beta,VC}$$

$$= \left\langle \phi_0 \middle| \sum_{n=0}^{2}\frac{i^n}{n!}\int \delta(t)d^4xd^4x_1 \ldots d^4x_n e^{-iq\cdot x}T[\mathcal{L}_r^{str}(x_1)\ldots\mathcal{L}_r^{str}(x_n)j_r^0(x)] \middle| \phi_0 \right\rangle_c^N$$

$$= 2\langle\phi_0| -\frac{1}{2}\int \delta(t)d^4xd^4x_1d^4x_2e^{-iq\cdot x} :\left(-\bar{\psi}(x_1)i\gamma^5\frac{\lambda_k}{F}\Phi_i(x_1)S(r)\psi_\alpha(x_1)\right)$$

$$\times\left(Q\bar{\psi}_\alpha(x)\gamma^0\psi_\beta(x)\right)\left(-\bar{\psi}_\beta(x_2)i\gamma^5\frac{\lambda_l}{F}\Phi_j(x_2)S(r)\psi(x_2)\right): |\phi_0\rangle.$$

(3.65)

For proton,

$$G_E^p(Q^2)\big|_{\alpha\beta,VC} = \frac{1}{(2\pi)^2F^2}\int_0^\infty dpp^2F1*F2*\ F3$$

$$\times\left[\begin{array}{l}\frac{1}{\omega_\pi(\vec{p})(\omega_\pi(\vec{p})+\Delta\varepsilon_\alpha)(\omega_\pi(\vec{p})+\Delta\varepsilon_\beta)}\\ -\frac{2}{\omega_K(\vec{p})(\omega_K(\vec{p})+\Delta\varepsilon_\alpha)(\omega_K(\vec{p})+\Delta\varepsilon_\beta)}\\ +\frac{1/3}{\omega_\eta(\vec{p})(\omega_\eta(\vec{p})+\Delta\varepsilon_\alpha)(\omega_\eta(\vec{p})+\Delta\varepsilon_\beta)}\end{array}\right] \qquad (3.66)$$

Where, $F1$, $F1$ and $F3$ are defined as below

$$F1 \equiv (-N_0N_\alpha)\int_0^\infty r^2dr\int_0^\pi sin\theta cos\theta d\theta\int_0^{2\pi}d\phi\, S(r) \qquad (3.67)$$

$$\times\left(f_0(r)g_\alpha(r)+f_\alpha(r)g_0(r)\right)\left(C_0Y_{0,0}\right)\left(A_1Y_{\alpha,0}\right)e^{iprcos\theta}$$

$$F2 \equiv \left(N_\beta N_\alpha\right)\int_0^\infty r^2dr\int_0^\pi sin\theta d\theta\int_0^{2\pi}d\phi\, S(r)$$

$$\times\left(f_\alpha(r)f_\beta(r)+g_\alpha(r)g_\beta(r)\right) \qquad (3.68)$$

$$\times\left(A_1Y^*_{\alpha,0}B_1Y_{\beta,0}+A_2Y^*_{\alpha,1}B_2Y_{\beta,1}\right)e^{i\sqrt{Q^2}rcos\theta}$$

$$F3 \equiv \left(-N_0N_\beta\right)\int_0^\infty r^2dr\int_0^\pi sin\theta cos\theta d\theta\int_0^{2\pi}d\phi\, S(r) \qquad (3.69)$$

$$\times\left(f_0(r)g_\beta(r)+f_\beta(r)g_0(r)\right)(C_0Y_{0,0})(B_1Y^*_{\beta,0})e^{-iprcos\theta}$$

For neutron,

$$G_E^n(Q^2)|_{\alpha\beta,VC} = \frac{1}{(2\pi)^2F^2}\int_0^\infty dpp^2\, F4*\ F5*F6 \qquad (3.70)$$

$$\times\left[\begin{array}{l}\frac{2}{\omega_\pi(\vec{p})(\omega_\pi(\vec{p})+\Delta\varepsilon_\alpha)(\omega_\pi(\vec{p})+\Delta\varepsilon_\beta)}\\ -\frac{2}{\omega_K(\vec{p})(\omega_K(\vec{p})+\Delta\varepsilon_\alpha)(\omega_K(\vec{p})+\Delta\varepsilon_\beta)}\end{array}\right]$$

where $F4, F5$ and $F6$ are defined as below,

$$F4 \equiv (-N_0N_\alpha)\int_0^\infty r^2dr\int_0^\pi sin\theta cos\theta d\theta\int_0^{2\pi}d\phi\, S(r) \qquad (3.71)$$
$$\times\left(f_0(r)g_\alpha(r)+f_\alpha(r)g_0(r)\right)\left(C_0Y_{0,0}\right)\left(A_1Y_{\alpha,0}\right)e^{iprcos\theta}$$

$$F5 \equiv \left(N_\beta N_\alpha\right)\int_0^\infty r^2dr\int_0^\pi sin\theta d\theta\int_0^{2\pi}d\phi\, S(r)$$
$$\times\left(f_\alpha(r)f_\beta(r)+g_\alpha(r)g_\beta(r)\right) \qquad (3.72)$$
$$\times\left(A_1Y^*_{\alpha,0}B_1Y_{\beta,0}+A_2Y^*_{\alpha,1}B_2Y_{\beta,1}\right)e^{i\sqrt{Q^2}rcos\theta}$$

$$F6 \equiv \left(-N_0N_\beta\right)\int_0^\infty r^2dr\int_0^\pi sin\theta cos\theta d\theta\int_0^{2\pi}d\phi\, S(r)$$
$$\times\left(f_0(r)g_\beta(r)+f_\beta(r)g_0(r)\right) \qquad (3.73)$$
$$\times(C_0Y_{0,0})(B_1Y^*_{\beta,0})e^{-iprcos\theta}$$

The express for the $G_M^N(Q^2)$ are,

$$\chi^\dagger_{N_{s'}}\frac{i\vec{\sigma}_N\times\vec{q}}{2M_N}\chi_{N_s}G_M^N(Q^2)|_{\alpha\beta,VC}$$
$$=\left\langle\phi_0\left|\sum_{n=0}^2\frac{i^n}{n!}\int\delta(t)d^4xd^4x_1\dots d^4x_ne^{-iq\cdot x}T[\mathcal{L}_r^{str}(x_1)\dots\mathcal{L}_r^{str}(x_n)\vec{j}_r(x)]\right|\phi_0\right\rangle_c^N$$
$$=2\langle\phi_0|-\frac{1}{2}\int\delta(t)d^4xd^4x_1d^4x_2e^{-iq\cdot x}:\left(-\bar{\psi}(x_1)i\gamma^5\frac{\lambda_k}{F}\Phi_i(x_1)S(r)\psi_\alpha(x_1)\right)$$
$$\times\left(Q\bar{\psi}_\alpha(x)\vec{\gamma}\psi_\beta(x)\right)\left(-\bar{\psi}_\beta(x_2)i\gamma^5\frac{\lambda_l}{F}\Phi_j(x_2)S(r)\psi(x_2)\right):|\phi_0\rangle$$

(3.74)

For proton;

$$G_M^p(Q^2)\big|_{\alpha\beta,VC} = \frac{2m_N i}{(2\pi)^2F^2}\int_0^\infty dpp^2\ F7 *\ F8 \times F9\left[\begin{array}{l}\frac{1/9}{\omega_\pi(\vec{p})(\omega_\pi(\vec{p})+\Delta\varepsilon_\alpha)\left(\omega_\pi(\vec{p})+\Delta\varepsilon_\beta\right)} \\ +\frac{2/9}{\omega_K(\vec{p})(\omega_K(\vec{p})+\Delta\varepsilon_\alpha)\left(\omega_K(\vec{p})+\Delta\varepsilon_\beta\right)} \\ -\frac{1/9}{\omega_\eta(\vec{p})\left(\omega_\eta(\vec{p})+\Delta\varepsilon_\alpha\right)\left(\omega_\eta(\vec{p})\ +\Delta\varepsilon_\beta\right)}\end{array}\right] \tag{3.75}$$

Where $F7, F8,$ and $F9$ are defined as below,

$$F7 \equiv (-N_0N_\alpha)\int_0^\infty r^2dr\int_0^\pi sin\theta cos\theta d\theta\int_0^{2\pi}d\phi\, S(r) \times \left(f_0(r)g_\alpha(r)+f_\alpha(r)g_0(r)\right)\left(C_0Y_{0,0}\right)\left(A_1Y_{\alpha,0}\right)e^{iprcos\theta} \tag{3.76}$$

$$F8 \equiv \left(N_\beta N_\alpha\right)\int_0^\infty r^2dr\int_0^\pi sin\theta cos\theta d\theta\int_0^{2\pi}d\phi\, S(r) \times \left(f_\alpha(r)g_\beta(r)+g_\alpha(r)f_\beta(r)\right)\frac{1}{\sqrt{Q^2}}\left(A_1Y_{\alpha,0}^*B_1Y_{\beta,0} - A_2Y_{\alpha,1}^*B_2Y_{\beta,1}\right)e^{i\sqrt{Q^2}rcos\theta} \tag{3.77}$$

$$F9 \equiv \left(-N_0N_\beta\right)\int_0^\infty r^2dr\int_0^\pi sin\theta cos\theta d\theta\int_0^{2\pi}d\phi\, S(r) \times \left(f_0(r)g_\beta(r)+f_\beta(r)g_0(r)\right)(C_0Y_{0,0})(B_1Y_{\beta,0}^*)e^{-iprcos\theta} \tag{3.78}$$

$$G_M^n(Q^2)|_{\alpha\beta,VC} = \frac{2m_N i}{(2\pi)^2F^2}\int_0^\infty dpp^2F10 * F11 * F12 \times\left[\begin{array}{l}\frac{-4/9}{\omega_\pi(\vec{p})(\omega_\pi(\vec{p})+\Delta\varepsilon_\alpha)\left(\omega_\pi(\vec{p})+\Delta\varepsilon_\beta\right)} \\ +\frac{2/9}{\omega_K(\vec{p})(\omega_K(\vec{p})+\Delta\varepsilon_\alpha)\left(\omega_K(\vec{p})+\Delta\varepsilon_\beta\right)} \\ +\frac{2/27}{\omega_\eta(\vec{p})\left(\omega_\eta(\vec{p})+\Delta\varepsilon_\alpha\right)\left(\omega_\eta(\vec{p})\ +\Delta\varepsilon_\beta\right)}\end{array}\right] \tag{3.79}$$

where $F10, F11,$ and $F12$ are defined as below,

$$F10 \equiv (-N_0N_\alpha)\int_0^\infty r^2dr\int_0^\pi sin\theta cos\theta d\theta\int_0^{2\pi}d\phi\, S(r) \times \left(f_0(r)g_\alpha(r)+f_\alpha(r)g_0(r)\right)\left(C_0Y_{0,0}\right)\left(A_1Y_{\alpha,0}\right)e^{iprcos\theta} \tag{3.80}$$

$$F11 \equiv \left(N_\beta N_\alpha\right)\int_0^\infty r^2dr\int_0^\pi sin\theta cos\theta d\theta\int_0^{2\pi}d\phi\, S(r) \times \left(f_\alpha(r)g_\beta(r)+g_\alpha(r)f_\beta(r)\right)\frac{1}{\sqrt{Q^2}}\left(A_1Y_{\alpha,0}^*B_1Y_{\beta,0} - A_2Y_{\alpha,1}^*B_2Y_{\beta,1}\right)e^{i\sqrt{Q^2}rcos\theta} \tag{3.81}$$

$$F12 \equiv (-N_0 N_\beta) \int_0^\infty r^2 dr \int_0^\pi sin\theta cos\theta d\theta \int_0^{2\pi} d\phi\, S(r) \\ \times \left(f_0(r) g_\beta(r) + f_\beta(r) g_0(r)\right) (C_0 Y_{0,0})(B_1 Y^*_{\beta,0}) e^{-iprcos\theta} \tag{3.82}$$

Finally, the Clebsch-Gordan coefficients are $A_1 = \left(l_\alpha 0 \frac{1}{2}\frac{1}{2} \middle| j_\alpha \frac{1}{2}\right)$, $A_2 = \left(l_\alpha 1 \frac{1}{2} - \frac{1}{2} \middle| j_\alpha \frac{1}{2}\right)$, $B_1 = \left(l_\beta 0 \frac{1}{2}\frac{1}{2} \middle| j_\beta \frac{1}{2}\right)$, and $B_2 = \left(l_\beta 1 \frac{1}{2} - \frac{1}{2} \middle| j_\beta \frac{1}{2}\right)$.

5. The meson-in-flight (MF) diagram:

$$\chi^\dagger_{N_{s'}} \chi_{N_s} G_E^N(Q^2)|_{MF}$$
$$= \left\langle \phi_0 \middle| \sum_{n=0}^{2} \frac{i^n}{n!} \int \delta(t) d^4x d^4x_1 \ldots d^4x_n e^{-iq\cdot x} T[\mathcal{L}_r^{str}(x_1) \ldots \mathcal{L}_r^{str}(x_n) j_\Phi^0(x)] \middle| \phi_0 \right\rangle_c^N$$
$$= 2\langle \phi_0 | -\frac{1}{2} \int \delta(t) d^4x d^4x_1 d^4x_2 e^{-iq\cdot x} : \left(-\bar{\psi}(x_1) i\gamma^5 \frac{\lambda_k}{F} \Phi_k(x_1) S(r) \psi_0(x_1)\right)$$
$$\times \left(-\bar{\psi}_\alpha(x_2) i\gamma^5 \frac{\lambda_l}{F} \Phi_l(x_2) S(r) \psi(x_2)\right) \times \left(f_{3ij} + \frac{f_{8ij}}{\sqrt{3}}\right) \Phi_i(x) \frac{\partial \Phi_j(x)}{\partial t} : |\phi_0\rangle. \tag{3.83}$$

For this diagram, we cannot apply excited states quark propagators, and there is no contraction between the quarks in this case. After using some algebras, we found that the integral corresponding to the energy part is zero, makes the value of $G_E^N(Q^2)|_{MF}$ becomes zero identically for proton and neutron.

$$G_E^N(Q^2)|_{MF} = 0, \tag{3.84}$$

Finally, the analytical expressions for the magnetic form factor of the MF diagram are

$$\chi^\dagger_{N_{s'}} \frac{i\vec{\sigma}_N \times \vec{q}}{2M_N} \chi_{N_s} G_M^N(Q^2)|_{MF}$$
$$= \left\langle \phi_0 \middle| \sum_{n=0}^{2} \frac{i^n}{n!} \int \delta(t) d^4x d^4x_1 d^4x_2 e^{-iq\cdot x} T[\mathcal{L}_r^{str}(x_1) \mathcal{L}_r^{str}(x_2) \vec{j}_\Phi(x)] \middle| \phi_0 \right\rangle_c^N$$
$$= 2\langle \phi_0 | -\frac{1}{2} \int \delta(t) d^4x d^4x_1 d^4x_2 e^{-iq\cdot x} : \left(-\bar{\psi}(x_1) i\gamma^5 \frac{\lambda_k}{F} \Phi_k(x_1) S(r) \psi_0(x_1)\right)$$
$$\times \left(-\bar{\psi}_\alpha(x_2) i\gamma^5 \frac{\lambda_l}{F} \Phi_l(x_2) S(r) \psi(x_2)\right) \times \left(f_{3ij} + \frac{f_{8ij}}{\sqrt{3}}\right) \Phi_i(x) \nabla \Phi_j(x) : |\phi_0\rangle$$
$$= \frac{-i}{(2\pi)^3 F^2} \langle \phi_0 | b^\dagger_{0,\alpha} b^\dagger_{0,\gamma} \int d^3p\, \vec{p} \times \frac{(\vec{\sigma}\cdot(\vec{p}+\vec{q}))}{|\vec{p}+\vec{q}|} \cdot \frac{(\vec{\sigma}\cdot\vec{p})}{|\vec{p}|}$$
$$\times \int d^3x_1 \left(\bar{u}_0(\vec{x}_1) i\gamma^5 S(x_1) u_\alpha(\vec{x}_1) e^{i(\vec{p}+\vec{q})\cdot\vec{x}_1}\right)$$

$$\times\left(\int d^3x_2\,\bar{u}_\alpha(\vec{x}_2)i\gamma^5 S(x_2)u_0(\vec{x}_2)e^{-i\vec{p}\cdot\vec{x}_2}\right)$$
$$\times\left(f_{3ij}+\frac{f_{8ij}}{\sqrt{3}}\right)\frac{\lambda_i^{\alpha\beta}\lambda_j^{\gamma\delta}}{\omega_\phi^2(\vec{p}+\vec{q})\omega_\phi^2(\vec{p})}b_{0,\delta}b_{0,\beta}|\phi_0\rangle. \tag{3.85}$$

For proton $N = p$;

$$G_M^p(Q^2)\big|_{MF} = \frac{m_N}{(2\pi)^2F^2}\int_0^\infty dpp^3\int_{-1}^1 dx\frac{1-x^2}{\sqrt{p^2+Q^2+2p\sqrt{Q^2}x}} * \left[\frac{4}{\omega_\pi{}^2(\vec{p})\omega_\pi{}^2(\vec{p}+\vec{q})}\right]$$
$$\times(-N_0N_0)\int_0^\infty r^2dr\int_0^\pi sin\theta cos\theta d\theta\int_0^{2\pi}d\phi\, S(r)\big(2f_0(r)g_0(r)\big)\big(C_0Y_{0,0}\big)^2 e^{i|\vec{p}+\vec{q}|rcos\theta}$$
$$\times(-N_0N_0)\int_0^\infty r^2dr\int_0^\pi sin\theta cos\theta d\theta\int_0^{2\pi}d\phi\, S(r)\big(2f_0(r)g_0(r)\big)\big(C_0Y_{0,0}\big)^2 e^{-iprcos\theta}. \tag{3.86}$$

For neutron $N = n$;

$$G_M^n(Q^2)\big|_{MF} = \frac{m_N}{(2\pi)^2F^2}\int_0^\infty dpp^3\int_{-1}^1 dx\frac{1-x^2}{\sqrt{p^2+Q^2+2p\sqrt{Q^2}x}} * \left[\frac{-4}{\omega_\pi{}^2(\vec{p})\omega_\pi{}^2(\vec{p}+\vec{q})}\right]$$
$$\times(-N_0N_0)\int_0^\infty r^2dr\int_0^\pi sin\theta cos\theta d\theta\int_0^{2\pi}d\phi\, S(r)\big(2f_0(r)g_0(r)\big)\big(C_0Y_{0,0}\big)^2 e^{i|\vec{p}+\vec{q}|rcos\theta}$$
$$\times(-N_0N_0)\int_0^\infty r^2dr\int_0^\pi sin\theta cos\theta d\theta\int_0^{2\pi}d\phi\, S(r)\big(2f_0(r)g_0(r)\big)\big(C_0Y_{0,0}\big)^2 e^{-iprcos\theta} \tag{3.87}$$

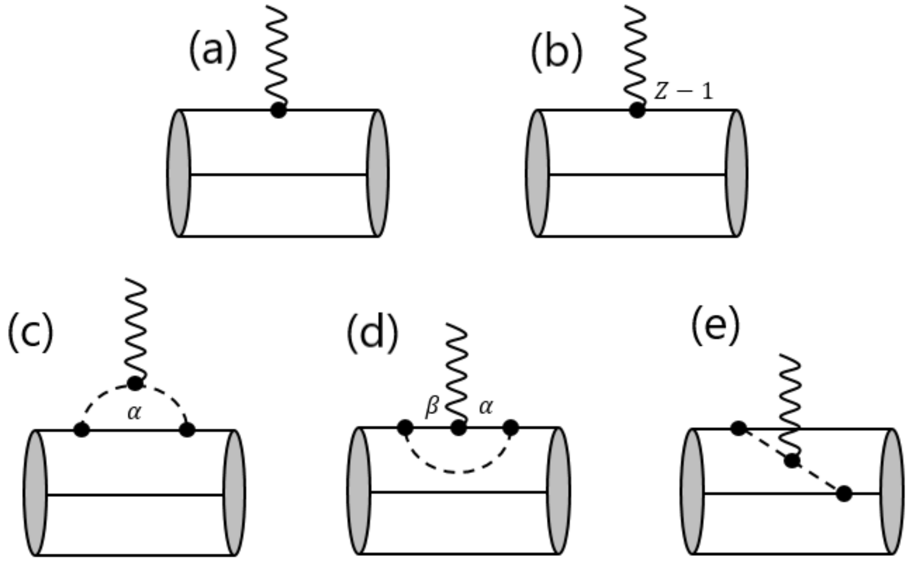

FIGURE 1 The Feynman diagrams for the Sachs form factors in the PCQM: (a) 3q-core, (b) counter-term, (c) meson-cloud, (d) vertex-correction, and (e) meson-in-flight.

CHAPTER 4
RESULTS AND DISCUSSION

We study the electromagnetic form factors of the nucleon in the framework of the PCQM. We emphasize our analysis in the region of low Q^2, from 0.0 up to 0.4 GeV^2, where the meson cloud contributions are expected to play an essential role in the electromagnetic form factors. In our approach, we study the effect of the inclusion of the excited quark states on the quark propagator. We include two low-lying excited states which are $1p_{1/2}, 1p_{3/2}, 1d_{3/2}, 1d_{5/2}$ and $2s_{1/2}$ states. For simplicity, we start by setting the value of the parameter $R = 0.60\ \mathrm{fm}$, as previously done before in the PCQM. The appropriate parameter ρ is fixed by considering the experimental data of the magnetic moments of the nucleon, μ_p and μ_n. We found that the value of ρ between 0.51 and 0.59 can reproduce the experimental values of μ_p and μ_n. Table 3 shows our results for some properties of the nucleon, see also (17).

The μ_p is best described with $\rho = 0.51$, whereas the best result for μ_n refers to the value of $\rho = 0.59$. Therefore, in Table 3, we also report the numerical calculations at the central value of $\rho = 0.55$.

TABLE 3 The results for the nucleon magnetic moments (in units of the nuclear magneton), the charge and the magnetic radii for the nucleon. The experimental data are taken from the PDG (18). Note that we fixed R = 0.6 fm.

	$\rho = 0.51$	$\rho = 0.55$	$\rho = 0.59$	Exp (18)
μ_p	2.7913	2.728	2.6716	2.793
μ_n	-2.0278	-1.958	-1.898	-1.913

Table 3 (Continued)

	$\rho = 0.51$	$\rho = 0.55$	$\rho = 0.59$	Exp (18)
$< r_E >_p$	0.8915	0.8956	0.900	0.8751, 0.84087
$< r_E^2 >_n$	-0.1425	-0.1129	-0.0537	-0.1161
$< r_M >_p$	1.010	0.965	0.8779	0.851
$< r_M >_n$	0.8447	0.9028	0.8335	0.864

Obviously, the modification of the quark propagator significantly improved the results. Consult (17) for an additional discussion.

Detail analysis of our results for the parameters $\rho = 0.55$ and $R = 0.60$ fm are shown in Table 4. Their values with quark excited states are improved significantly and become closer to the experimental data in the range error of ± 2 %. Furthermore, the excited states contribute the most in the neutron charge radius, which is about 31 %. The neutron charge radius value is very close and does not exceed the experimental data, whereas the proton charge radius with the quark excited state is also nearly equal to the experiment, but a bit exceeds the experimental data (0.8751) around $+2$ %. The ground state contribution is only 67 % compared with the experimental data (-0.1161). For magnetic radii, the ground state contribution to the nucleon is at the level of 80 % comparing to the experimental data. Their values become much more prominent when we include the excited states, but it seems to be a bit excessive. The neutron magnetic radius is 4% bigger, while the proton magnetic radius is about 13% bigger than the experimental data.

TABLE 4 The separation of contributions from the ground state and the excited states to the nucleon properties for R = 0.6 fm and rho = 0.55.

	The ground state contribution	The excited states contribution	Total contribution	Percentage contribution of the excited states	Experiment (18)
μ_p	2.5105	0.2175	2.728	9%	2.793
μ_n	-1.778	-0.18	-1.958	10%	-1.913
$< r_E >_p$	0.8537	0.0419	0.8956	5%	0.8751, 0.84087
$< r_E^2 >_n$	-0.07785	-0.03505	-0.1129	31%	-0.1161
$< r_M >_p$	0.6835	0.2819	0.965	29%	0.851
$< r_M >_n$	0.6732	0.2296	0.9028	25%	0.864

Table 5 shows our calculation results of each diagram for the set parameters $\rho = 0.55$ and $R = 0.60\ \mathrm{fm}$, along with its contribution in detail. We found that the three-quark diagram (3q core) contributes the most in proton charge radius (87.4%), the magnetic moment of the proton (87.3%) and neutron (81.1%). However, the values decrease in proton magnetic radius (50.1%) and neutron magnetic radius (43.8%), which becomes the same level of contribution as from the meson-cloud diagram. However, it contributes nothing to the charge neutron radius. The counter-term (CT) diagram always contributes the opposite sign to the three-quark diagram (except the neutron charge radius, which its value is equal to zero). The percentage of meson cloud diagram contribution is also reported in the table. It contributes mostly to the neutron charge radius (113%) and shows a crucial effect on that value. However, its contribution decreases significantly in the proton magnetic radius (50.1%) and neutron magnetic radius (40.0%), and becomes only 10-20% in the proton charge radius, magnetic moments of proton and neutron. The meson-in-flight (MF) diagram comes in a third of the ranking of contributions. It contributes 29.2% in neutron magnetic radius, 26.6% in proton magnetic radius and roughly 10% for nucleon magnetic moments. From our calculations, it contributes 0% in proton and neutron charge radii. The Vertex correction diagram has only a few percentage contributions but plays an important role, especially in the neutron charge radius (because all the contributions come from the vertex-correction diagram and the meson-cloud diagrams). It contributes less to both magnetic moments of proton and neutron.

TABLE 5 Contribution of each diagram to the nucleon electromagnetic properties for R = 0.6 fm and rho = 0.55.

Diagram / Properties	Meson Cloud	Vertex Correction	Meson in flight	3quark core	Counter term	Total
μ_p	0.3426	-0.0072	0.1987	2.3816	-0.1882	2.728
	12.6%	-0.3%	7.3%	87.3%	-6.9%	

TABLE 5 (Continued)

Diagram / Properties	Meson Cloud	Vertex Correction	Meson in flight	3quark core	Counter term	Total
μ_n	-0.3141	0.0165	-0.1987	-1.5877	0.1255	-1.958
	16.0%	-0.8%	10.1%	81.1%	-6.4%	
$< r_E^2 >_p$	0.1401	0.0079	0.000	0.7007	-0.0467	0.8021
	17.5%	1.0%	0.0%	87.4%	-5.8%	
$< r_E^2 >_n$	-0.1285	0.0156	0.000	0.000	0.000	- 0.1129
	113.8%	-13.8%	0.0%	0.0%	0.0%	
$< r_M^2 >_p$	0.467	-0.1331	0.248	0.46712	-0.1168	0.932
	50.1%	-14.3%	26.6%	50.1%	-12.5%	
$< r_M^2 >_n$	0.334	-0.066	0.238	0.357	-0.048	0.815
	41.0%	-8.1%	29.2%	43.8%	-5.9%	

In addition, the nucleon electromagnetic form factors up to $Q^2 = 0.4\,\mathrm{GeV}^2$ are presented in Fig.2 to Fig. 5. In Fig.2, we compare our result for $G_E^p(Q^2)$ to the dipole fit $G_D(Q^2)$,

$$G_D(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{0.71}\right)^2} \tag{4.1}$$

Similarly, our results for $G_M^p(Q^2)$ and $G_M^n(Q^2)$ are shown in Fig.4 and Fig.5 in comparison to the $\mu_p G_D(Q^2)$ and $\mu_n G_D(Q^2)$, respectively. Finally, Fig.3 shows our result for $G_E^n(Q^2)$ in comparison to the lattice QCD calculation and the experimental data.

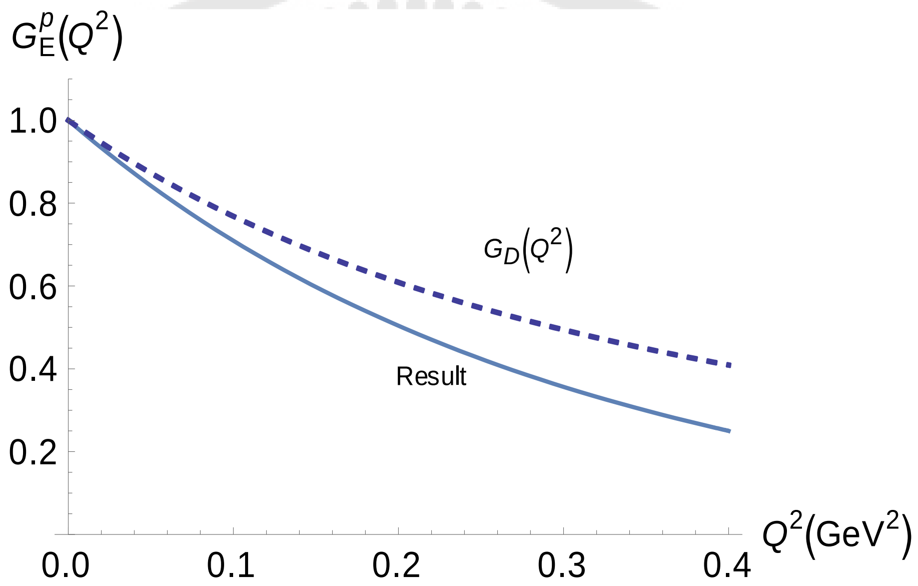

FIGURE 2 Proton charge form factor for R = 0.6 fm and rho = 0.55.

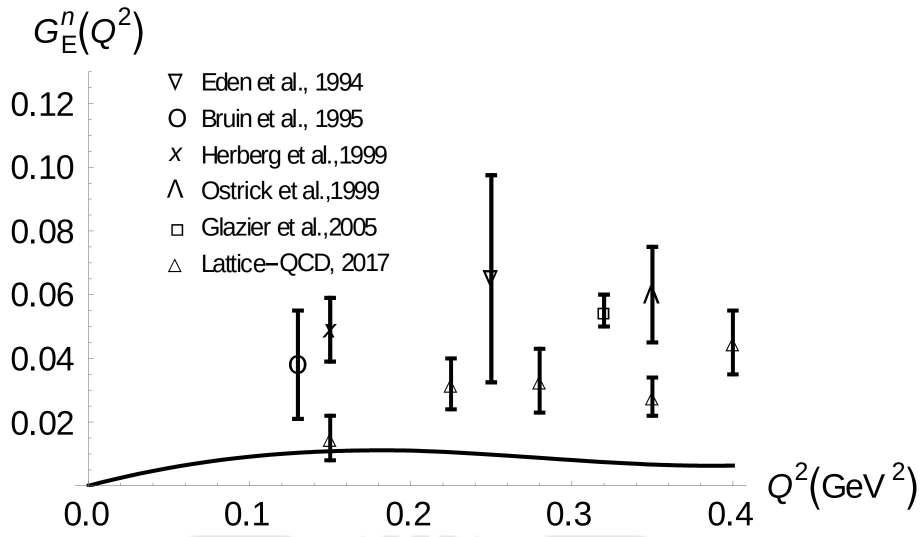

FIGURE 3 Neutron charge form factor for R = 0.60 fm and rho = 0.55. The experimental data are taken from (19) and the lattice data are taken from Ref. (20-24).

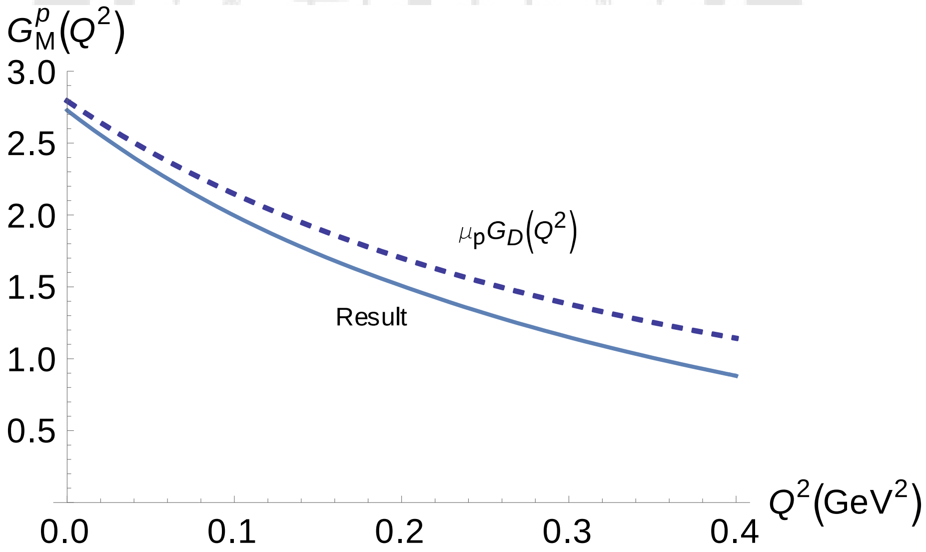

FIGURE 4 Proton magnetic form factor for R = 0.6 fm and rho = 0.55.

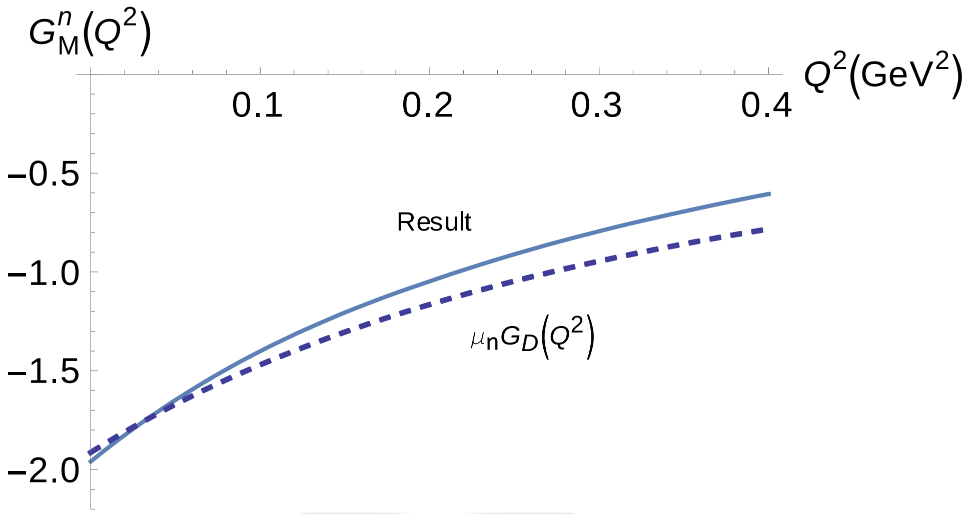

FIGURE 5 Neutron magnetic form factor for R = 0.6 fm and rho = 0.55.

Our calculation results for $G_E^p(Q^2)$, $G_M^p(Q^2)$ and $G_M^p(Q^2)$ are consistent with the data from dipole fit quite well, especially at the low Q^2. However, the value of $G_E^n(Q^2)$ is still smaller when compared with the experimental. There is one point that can reach the lattice QCD calculation at $Q^2 = 0.15\ \mathrm{GeV}^2$. Our calculation results of $G_E^p(Q^2)$ and $G_E^n(Q^2)$ at $Q^2 = 0$ are equal to one (the proton charge) and zero (the neutron charge), respectively, as expected. And the shape of $G_E^p(Q^2)$, $G_M^p(Q^2)$ and $G_M^p(Q^2)$ when $Q^2 > 0$ are in the same tendency with theoretical calculation results from $G_D(Q^2)$.

Furthermore, we investigate the energy level of the quark in the excited states, up to the fifth excited states. We try to modify the parameters of the model a bit and calculate the energy eigenstate with a new value of $\rho = 0.65$, and $R = 0.60$ fm. The energy of the quark in the ground state and in the other states up to the fifth states are shown in Table 6.

TABLE 6 The energy levels and notations of the quark states for R = 0.6 fm and rho = 0.55.

States	Notation	$\Delta\mathcal{E}_{\alpha}$ (GeV)
Ground state	$1s_{1/2}$	0
1st excited states	$1p_{1/2}, 1p_{3/2}$	0.239317
2nd excited states	$1d_{3/2}, 1d_{5/2}, 2s_{1/2}$	0.448240
3rd excited states	$1f_{5/2}, 1f_{7/2}, 2p_{1/2}, 2p_{3/2}$	0.638042
4th excited states	$1g_{7/2}, 1g_{9/2}, 2d_{3/2}, 2d_{5/2}, 3s_{1/2}$	0.814244
5th excited states	$1h_{9/2}, 1h_{11/2}, 2f_{5/2}, 2f_{7/2}, 3p_{1/2}, 3p_{3/2}$	0.980063

In Table 7, the results of the nucleon magnetic moments after the inclusion up to the fifth excited states to the quark propagators with a new value of $\rho = 0.65$ and the value of $R = 0.60$ fm give a better agreement to the experimental data, which is 2.7514 for the proton magnetic moment and -1.934 for the neutron magnetic moment. This value of ρ is quite different from the previous work, which is $\rho = 0.39$ in Ref. (2). We found that the suitable value of ρ depends on the number of excited states that included in the calculation. The ground state contributes at the same level to both proton and neutron, which is about 90%, while the excited states contribute about 10%. Moreover, each excited state from the first excited states to the fifth excited states contributes nearly the same percentage on the nucleon magnetic moments, about 2% each. From these results, we can express the importance of those higher excited states of quark propagators.

TABLE 7 Results for the magnetic moments of the nucleon with the modified quark propagator (up to the fifth states) for R = 0.6 fm and rho = 0.55.

	μ_p		μ_n	
States of quark propagator	Value	%	Value	%
Ground state only	2.4592	89%	-1.7021	88%
1st excited state	0.0698	3%	-0.06104	3%
2nd excited state	0.0536	2%	-0.04306	2%
3rd excited state	0.0542	2%	-0.0412	2%
4th excited state	0.0592	2%	-0.0446	2%
5th excited state	0.0554	2%	-0.042	2%
Total (GS+ up to 5th excited states)	2.7514		-1.934	
Experiment (18)	2.793		-1.913	

Finally, in Table 8, we present the contributions, up to the fifth states, of each diagram to the values of the nucleon magnetic moments. The 3q core diagram plays a crucial role in the magnetic moments of nucleons. It contributes 82.5% for proton and 78.2% for neutron, respectively. The mesons also provide significantly high contributions, which is 17.5% for proton and 21.8% for neutron in magnitude. This implies that we cannot neglect the effects from the surrounding mesons and the mesons those involved in the strong interaction between the valence quarks inside the nucleon. In the mesonic effect, we found that the meson- cloud diagram gives the most significant contribution, whereas the vertex correction gives the smallest contribution. The contribution from the meson-in-flight diagram is nearly equal to the contribution from the counter-term diagram. Moreover, the sign of the counter-terms is always the opposite of the 3q core diagram. These results are in the same tendency of the previous results, with the truncation to the ground state quark. It shows the consistency between the including and not including the quark excited states into the quark propagator.

TABLE 8 Contribution of each diagram to the magnetic moments of the nucleon with the modified quark propagator (up to the fifth states) for R = 0.6 fm and rho = 0.55.

	Type	3q (LO)	Meson effects					Total
			3q Next to LO	Counter term	Meson cloud	Vertex correction	Meson in flight	
Proton	Value	2.2698	0.2489	-0.1621	0.282	-0.0052	0.1181	2.751
	%	82.5%	17.5%					100%
Neutron	Value	-1.5132	-0.1659	0.1081	-0.2562	0.0112	-0.1181	-1.934
	%	78.2%	21.8%					100%

CHAPTER 5
SUMMARY AND CONCLUSION

We have investigated the electromagnetic properties of the nucleon and the Sachs form factors in the low Q^2 domain, up to 0.4 GeV^2. The first two quark excited states have been added to the quark propagator for the studying of the nucleon properties. We summarise our main results as follows.

At $Q^2 = 0$ the excited states contribute significantly to the nucleon magnetic moments, which made the calculation results better and closer to the experimental data. Furthermore, for $Q^2 > 0$ the excited states contribute around 30-40% on average. Especially in the neutron charge radius, they contribute 45%, which made the calculation tightly fit the experimental data. These results can prove the necessity of excited states quark propagators for nucleonic sector electromagnetic properties.

The new appropriate set of parameters for the Gaussian ansatz quark wave function are $\rho = 0.55, R = 0.60$. These parameters are variable with respect to the energy of the valence quark and directly affect the quark wave function.

At $Q^2 = 0$, the most contribution to the nucleon magnetic moments coms from the 3q-core diagram. However, at $Q^2 > 0$ the 3q-core diagram contribution drops drastically to around 50%, while the contribution from the meson-cloud diagram significantly arises to the same level of contribution coming from the three-quark diagram.

From Fig.2 to Fig.5, our results of the $G_E^p(Q^2)$, $G_M^p(Q^2)$ and $G_M^p(Q^2)$ at low-Q^2 are less than the dipole fit, because the Gaussian Ansatz form of the wave function has been used in our calculation. Besides, the value of $G_E^n(Q^2)$ is still small when compared to the experimental data.

Furthermore, in order to consider the effects of the higher excited states of quark propagators on the value of magnetic moments of the nucleon, we also include the third, the fourth and the fifth excited states to the quark propagator. The new value of $\rho = 0.65$ seems to improve our results. Our results of the nucleon magnetic moments with a new value of $\rho = 0.65$ give a better agreement to the experimental data, which is 2.7514 for the proton magnetic moment and -1.934 for the neutron magnetic moment.

As indicated before in previous works of this model, the 3q-core diagram plays a crucial role in the case of the nucleon magnetic moments, and they contribute about 70%. The rest 30% comes from the interplays between the meson cloud effect and the quark propagator in the excited states.

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Appendix

Appendix A

The Pauli matrices and Dirac Gamma matrices and some properties

The Pauli matrices and Dirac Gamma matrices and some properties

The Dirac equation is given below,

$$\mathcal{L}(x) = \bar{\psi}(x)\left[i\gamma^{\mu}\partial_{\mu} - m\right]\psi(x)$$

We can obtain the equation of motion from the Lagrangian, the so-call Dirac equation.

$$\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi(\vec{x},t) = i\gamma^{0}\frac{\partial\psi(\vec{x},t)}{\partial t} + i\vec{\gamma}\cdot\vec{\nabla}\psi(\vec{x},t) = 0$$

The Dirac gamma matrices are four unitary and traceless 4×4 matrices:

$$\gamma^{0} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\ ,\gamma^{i} = \begin{pmatrix} 0 & \sigma^{i} \\ -\sigma^{i} & 0 \end{pmatrix}.$$

The Pauli matrices are Hermitian, unitary and traceless 2×2 matrices:

$$\sigma^{1} \equiv \sigma_{x} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\ ,\ \sigma^{2} \equiv \sigma_{y} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$

And

$$\sigma^{3} = \sigma_{z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

We can expand the above equation to obtain the Dirac equation in terms of $\vec{\alpha}$ matrices and β matrix.

$$\left(-i\vec{\alpha}\cdot\vec{\nabla} + \beta m\right)\psi(x,t) = i\frac{\partial\psi(\vec{x},t)}{\partial t}$$

or in terms of momentum,

$$(\vec{\alpha}\cdot\vec{p} + \beta m)\psi(x,t) = i\frac{\partial\psi(\vec{x},t)}{\partial t}$$

By definition, the $\vec{\alpha}$ matrices and β matrix are 4×4 matrices, given by

$$\alpha^{i} = \begin{pmatrix} 0 & \sigma^{i} \\ \sigma^{i} & 0 \end{pmatrix}\ ,\quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Some important properties of Dirac gamma matrices are given below,

(i) $\gamma^{0} = \beta$,

(ii) $\gamma^{i} = \beta\alpha^{i}$,

(iii) $\{\gamma^{\mu},\gamma^{\nu}\} = 2g^{\mu\nu}$,

(iv) $\{\gamma_{\mu},\gamma^{\nu}\} = 2\delta_{\mu}^{\nu}$,

(v) $\left(\gamma^{i}\right)^{2} = -1$, $(\gamma^{0})^{2} = 1$,

(vi) $(\gamma^i)^\dagger = -\gamma^i$, $(\gamma^0)^\dagger = \gamma^0$.

Besides, there are frequently two combinations of gamma matrices in particle physics. The so-called sigma-matrices are defined as below,

$$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$$

Another important matrix is γ^5 defined as

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \gamma_5$$

and we can get

$$\gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

which is also a Hermitian matrix and have some important properties,

(i) $(\gamma^5)^\dagger = \gamma^5$,

(ii) $(\gamma^5)^2 = 1$,

(iii) $\{\gamma^5, \gamma^\mu\} = 0$.

Further, we introduce important properties of the Pauli matrices are given below,

(i) they satisfy the commutation relation

$$[\sigma^i, \sigma^j] = 2i\epsilon_{ijk}\sigma^k,$$

(ii) and the anti-commutation relation

$$\{\sigma^i, \sigma^j\} = 2\delta_{ij},$$

(iii) satisfy the product rule

$$\sigma^i\sigma^j = \delta_{ij} + i2\epsilon^{ijk}\sigma^k$$

(iv) For any two vectors $\overrightarrow{A}$ and $\overrightarrow{B}$, we can write in the form of

$$(\vec{\sigma}\cdot\overrightarrow{A})(\vec{\sigma}\cdot\overrightarrow{B}) = \overrightarrow{A}\cdot\overrightarrow{B} + i\overrightarrow{\sigma}\cdot(\overrightarrow{A}\times\overrightarrow{B})$$

$$(\vec{\sigma}\cdot(\vec{p}+\vec{q}))(\vec{\sigma}\cdot\vec{p}) = (\vec{p}+\vec{q})\cdot\vec{p} + i\overrightarrow{\sigma}\cdot((\vec{p}+\vec{q})\times\vec{p})$$

(v) operation in an intrinsic spin space of a two-body system

$$\begin{aligned}\vec{\sigma}(1)\cdot\vec{\sigma}(2) &= \sigma^1(1)\sigma^1(2) + \sigma^2(1)\sigma^2(2) + \sigma^3(1)\sigma^3(2) \\ &= \sigma_x(1)\sigma_x(2) + \sigma_y(1)\sigma_y(2) + \sigma_z(1)\sigma_z(2)\end{aligned}$$

For example,

$\vec{\sigma}(1)\cdot\vec{\sigma}(2)\left|-\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\right\rangle$

$=\sigma_x(1)\sigma_x(2)+\ \sigma_y(1)\sigma_y(2)$

$+\sigma_z(1)\sigma_z(2)\left|-\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\right\rangle$

$=\sigma_x(1)\sigma_x(2)\left|-\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\right\rangle$

$+\ \sigma_y(1)\sigma_y(2)\left|-\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\right\rangle$

$+\sigma_z(1)\sigma_z(2)\left|-\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\right\rangle$.

$=-\frac{1}{\sqrt{6}}[(\downarrow\uparrow+\uparrow\downarrow)\downarrow-2\uparrow\uparrow\uparrow]+-\frac{1}{\sqrt{6}}[(\downarrow\uparrow+\uparrow\downarrow)\downarrow+2\uparrow\uparrow\uparrow]$

$-\frac{1}{\sqrt{6}}[(-\uparrow\downarrow-\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]$.

Appendix B

Properties of SU (3) Groups

Properties of SU (3) Groups

In general, we usually use Gell-Mann matrices as the generators of 3×3 matrices (or $N = 3$). The Gell-Mann matrices are given below.

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},\ \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

The generators matrices λ_j obey the commutation relations and anti-commutation relation of the group.

$$\left[\frac{\lambda_i}{2}, \frac{\lambda_j}{2}\right] = i f_{ijk} \frac{\lambda_k}{2} \ , \quad i,j,k = 1,2,..8 \quad ,$$

$$\{\lambda_i, \lambda_j\} = \frac{4}{3}\delta_{ij} + 2d_{ijk}\lambda_k \ , \quad i,j,k = 1,2,..8 \ .$$

Where f_{ijk} are the structure constants of the $SU(3)$ group. They are anti-symmetric tensors, which values are given below.

$$f_{123} = 1, f_{458} = f_{678} = \frac{\sqrt{3}}{2},$$

$$f_{147} = f_{516} = f_{246} = f_{257} = f_{345} = f_{637} = \frac{1}{2}.$$

In our model, in the strong interaction Lagrangian term the Gell-Mann matrices will operate on the flavor part of the quark wave function, in which including the u-quark, d-quark and s-quark vectors. We can represent the u-quark, d-quark and s-quark in terms of basis vectors as below,

$$u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \ , \ d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \ , s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Here we show the results of the operations acted by Gell-Mann matrices on each flavor.

$$\lambda_1 u = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = d,$$

$$\lambda_1 d = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = u \ ,$$

$$\lambda_2 u = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = i\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = id,$$

$$\lambda_2 d = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = -i\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = -iu,$$

$$\lambda_3 u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = u,$$

$$\lambda_3 d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = -\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = -d\ ,$$

$$\lambda_4 u = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = s,$$

$$\lambda_4 d = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0\ ,$$

$$\lambda_5 u = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = i\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = is,$$

$$\lambda_5 d = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0\ ,$$

$$\lambda_6 u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0\ ,$$

$$\lambda_6 d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = s\ ,$$

$$\lambda_7 u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0\ ,$$

$$\lambda_7 d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = i\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = is\ ,$$

$$\lambda_8 u = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{3}}u\ ,$$

$$\lambda_8 d = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{3}}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{3}}d\ .$$

Wave function of a nucleon with spin-up, $|N\ ,\uparrow\rangle$

Appendix C

Calculation of matrix elements of the nucleons with spin up

Calculation of matrix elements of the nucleons with spin up

Nucleon, which is a proton or a neutron, consists of 3 valence quarks. The valence quarks are fermions, so they make the nucleon become a fermion too. All fermions obey the Pauli exclusion principle, which makes the nucleon wave function must be an anti-symmetric wave function. The wave function of a nucleon consists of several parts, there is the spatial part, spin part, flavor part, color part. Each part describe its characteristic, the spatial part describes the locations of the quarks. The spin part represents the spin of each quark. The flavor part represents the flavor of each quark. And the color part specifies the color of each quark.

$$\psi = \psi(space)\psi(spin)\psi(flavor)\psi(color)$$

When we evaluate the matrix element concerning with spin and flavor of quarks we can just pick up only spin part and flavor part of them in order to calculate the matrix element. The wave function of two parts become $SU(2) \times SU(3)$ and can be written as below,

$$\psi = \psi(spin)\psi(flavor),$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}|\phi_{MS}\chi_{MS} + \phi_{MA}\chi_{MA}\rangle,$$

$$\langle\psi| = \frac{1}{\sqrt{2}}\langle\phi_{MS}\chi_{MS} + \phi_{MA}\chi_{MA}|$$

Where ϕ_{MS} is the symmetric flavor wave function, ϕ_{MA} is the anti-symmetric flavor wave function. χ_{MS} is the spin-symmetric, and χ_{MA} is the spin anti-symmetric function. We can write neutron with spin up and proton with spin up in terms of ϕ_{MS} , ϕ_{MA} and χ_{MS} , χ_{MA} as below,

Neutron:

$$|\phi^n_{MS}\rangle = -\frac{1}{\sqrt{6}}[(ud+du)d - 2ddu], \quad |\phi^n_{MA}\rangle = \frac{1}{\sqrt{2}}(ud-du)d,$$

$$|\chi^n_{MS}\rangle = -\frac{1}{\sqrt{6}}[(\uparrow\downarrow + \downarrow\uparrow)\downarrow - 2\downarrow\downarrow\uparrow], \quad |\chi^n_{MA}\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow.$$

Proton:

$$|\phi^p_{MS}\rangle = \frac{1}{\sqrt{6}}[(ud+du)u - 2uud], \; |\phi^p_{MA}\rangle = \frac{1}{\sqrt{2}}(ud-du)u,$$

$$|\chi^p_{MS}\rangle = \frac{1}{\sqrt{6}}[(\uparrow\downarrow + \downarrow\uparrow)\uparrow - 2\uparrow\uparrow\downarrow], \; |\chi^p_{MA}\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow.$$

We evaluate the matrix elements which appear in the strong interaction Lagrangian of our model. The matrix elements compose of $SU(2) \times SU(3)$ those the spin operators and flavor operators are in the between of the initial state and final state. In our approach, the Pauli matrices $\vec{\sigma}$ are the spin operators which act on the spin part and Gell-Mann matrices $\lambda_i, i = 1,2..,8$ are flavor operators which act on the flavor part of the quark wave function.

The definition of the matrix element of the meson in flight diagram for mass shift inside the nucleon is defined by,

$$\left\langle \phi_0 \middle| b^{\dagger}_{0,\alpha} b^{\dagger}_{0,\gamma} \vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\gamma\delta}\, \lambda_j^{\alpha\beta} \lambda_j^{\gamma\delta} b_{0,\delta} \lambda_j^{\gamma\delta} b_{0,\beta} \middle| \phi_0 \right\rangle_c^N$$

$$= \left\langle N \uparrow \middle| \sum_{i \neq k} \vec{\sigma}_{\alpha\beta}(i) \cdot \vec{\sigma}_{\gamma\delta}(k)\, \lambda_j^{\alpha\beta}(i) \lambda_j^{\gamma\delta}(k) \middle| N \uparrow \right\rangle,$$

$$= 2\left\langle N \uparrow \middle| \vec{\sigma}(1) \cdot \vec{\sigma}(2)\, \lambda_j(1)\, \lambda_j(2) \middle| N \uparrow \right\rangle$$

$$+2\left\langle N \uparrow \middle| \vec{\sigma}(1) \cdot \vec{\sigma}(3)\, \lambda_j(1)\, \lambda_j(3) \middle| N \uparrow \right\rangle$$

$$+2\left\langle N \uparrow \middle| \vec{\sigma}(2) \cdot \vec{\sigma}(3)\, \lambda_j(2)\, \lambda_j(3) \middle| N \uparrow \right\rangle$$

From $SU(3)$, when $j = 1,2,3$ the above matrix elements are the contribution from pion mesons, while $j = 4,5,6,7$ the matrix elements are the contribution from kaon mesons, and the last one, $j = 8$ is the contribution from eta meson.

Here, we show the evaluation method of the matrix element which is the contribution come from pions ($j = 1,2,3$). First we evaluate for a neutron with spin up $|n\ \uparrow\rangle$.

$$\sum_{j=1}^{3} \left\langle n \uparrow \middle| \lambda_j(1)\, \lambda_j(2) \vec{\sigma}(1) \cdot \vec{\sigma}(2) \middle| n \uparrow \right\rangle$$

We show the method of calculation of matrix for $j = 1$, in details as below.

$$\langle n \uparrow | \lambda_1(1)\, \lambda_1(2) \vec{\sigma}(1) \cdot \vec{\sigma}(2) | n \uparrow \rangle$$

$$= \frac{1}{2} \langle \phi^n_{MS} | \lambda_1(1)\, \lambda_1(2) | \phi^n_{MS} \rangle \langle \chi^n_{MS} | \vec{\sigma}(1) \cdot \vec{\sigma}(2) | \chi^n_{MS} \rangle$$

$$+ \frac{1}{2} \langle \phi^n_{MS} | \lambda_1(1)\, \lambda_1(2) | \phi^n_{MA} \rangle \langle \chi^n_{MS} | \vec{\sigma}(1) \cdot \vec{\sigma}(2) | \chi^n_{MA} \rangle$$

$$+ \frac{1}{2} \langle \phi^n_{MA} | \lambda_1(1)\, \lambda_1(2) | \phi^n_{MS} \rangle \langle \chi^n_{MA} | \vec{\sigma}(1) \cdot \vec{\sigma}(2) | \chi^n_{MS} \rangle$$

$$+ \frac{1}{2} \langle \phi^n_{MA} | \lambda_1(1)\, \lambda_1(2) | \phi^n_{MA} \rangle \langle \chi^n_{MA} | \vec{\sigma}(1) \cdot \vec{\sigma}(2) | \chi^n_{MA} \rangle$$

We are able to separate the terms into pieces and evaluate all the pieces, and after that we sums the results totally.

1) $\langle \phi_{MS}^{n} | \, \lambda_1(1)\, \lambda_1(2) | \phi_{MS}^{n} \rangle$

$$= \left\langle -\frac{1}{\sqrt{6}}[(ud+du)d-2ddu] \right| \lambda_1(1)\,\lambda_1(2) \left| -\frac{1}{\sqrt{6}}[(ud+du)d-2ddu] \right\rangle$$

$$= \left\langle -\frac{1}{\sqrt{6}}[(ud+du)d-2ddu] \right| -\frac{1}{\sqrt{6}}[(du+ud)d-2uuu] \right\rangle$$

$$= \frac{1}{6} \times 2 = \frac{1}{3}$$

2) $\langle \phi_{MA}^{n} | \, \lambda_1(1)\, \lambda_1(2) | \phi_{MS}^{n} \rangle$

$$= \left\langle \frac{1}{\sqrt{2}}(ud-du)d \right| \lambda_1(1)\,\lambda_1(2) \left| -\frac{1}{\sqrt{6}}[(ud+du)d-2ddu] \right\rangle$$

$$= \left\langle \frac{1}{\sqrt{2}}(ud-du)d \right| -\frac{1}{\sqrt{6}}[(du+ud)d-2uuu] \right\rangle$$

$$= -\frac{1}{\sqrt{12}}(1-1) = 0$$

3) $\langle \phi_{MA}^{n} | \, \lambda_1(1)\, \lambda_1(2) | \phi_{MA}^{n} \rangle$

$$= \left\langle \frac{1}{\sqrt{2}}(ud-du)d \right| \lambda_1(1)\,\lambda_1(2) \left| \frac{1}{\sqrt{2}}(ud-du)d \right\rangle$$

$$= \left\langle \frac{1}{\sqrt{2}}(ud-du)d \right| \frac{1}{\sqrt{2}}(du-ud)d \right\rangle$$

$$= \frac{1}{2}(-1-1) = -1$$

The spin part of neutron are shown as below,

$$\left| \chi_{MS}^{n} \right\rangle = -\frac{1}{\sqrt{6}}[(\uparrow\downarrow + \downarrow\uparrow)\downarrow -2\downarrow\downarrow\uparrow] \;, |\chi_{MA}^{n}\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow$$

And then the matrix elements can be calculated as below.

1) $\langle \chi_{MS}^{n} | \vec{\sigma}(1)\cdot\vec{\sigma}(2) | \chi_{MS}^{n} \rangle$

$$= \left\langle -\frac{1}{\sqrt{6}}[(\uparrow\downarrow + \downarrow\uparrow)\downarrow -2\downarrow\downarrow\uparrow] \right| \vec{\sigma}(1)\cdot\vec{\sigma}(2) \left| -\frac{1}{\sqrt{6}}\begin{bmatrix} (\uparrow\downarrow + \downarrow\uparrow)\downarrow \\ -2\downarrow\downarrow\uparrow \end{bmatrix} \right\rangle$$

$$= \left\langle -\frac{1}{\sqrt{6}}[(\uparrow\downarrow + \downarrow\uparrow)\downarrow -2\downarrow\downarrow\uparrow] \right| \begin{matrix} \sigma_x(1)\sigma_x(2) \\ +\sigma_y(1)\sigma_y(2) \\ +\sigma_z(1)\sigma_z(2) \end{matrix} \left| -\frac{1}{\sqrt{6}}\begin{bmatrix} (\uparrow\downarrow + \downarrow\uparrow)\downarrow \\ -2\downarrow\downarrow\uparrow \end{bmatrix} \right\rangle$$

$$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

While the detail calculation are shown as below,

$$\left\langle -\frac{1}{\sqrt{6}}[(\uparrow\downarrow + \downarrow\uparrow)\downarrow -2\downarrow\downarrow\uparrow] \right| \sigma_x(1)\sigma_x(2) \left| -\frac{1}{\sqrt{6}}[(\uparrow\downarrow + \downarrow\uparrow)\downarrow -2\downarrow\downarrow\uparrow] \right\rangle$$

$$= \left\langle -\frac{1}{\sqrt{6}}[(\uparrow\downarrow + \downarrow\uparrow)\downarrow -2\downarrow\downarrow\uparrow] \right| -\frac{1}{\sqrt{6}}[(\downarrow\uparrow + \uparrow\downarrow)\downarrow -2\uparrow\uparrow\uparrow] \right\rangle$$

$$= \tfrac{1}{6} \times (2) = \tfrac{1}{3}.$$

$$\left\langle -\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\middle|\sigma_y(1)\sigma_y(2)\middle|-\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\right\rangle$$

$$=\left\langle -\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\middle|-\frac{1}{\sqrt{6}}[(\downarrow\uparrow+\uparrow\downarrow)\downarrow+2\uparrow\uparrow\uparrow]\right\rangle$$

$$=\frac{1}{6}\times(2)=\frac{1}{3}$$

$$\left\langle -\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\middle|\sigma_z(1)\sigma_z(2)\middle|-\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\right\rangle$$

$$=\left\langle -\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\middle|-\frac{1}{\sqrt{6}}[(-\uparrow\downarrow-\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\right\rangle$$

$$=\frac{1}{3}$$

2) $\langle\chi_{MS}^{n}|\vec{\sigma}(1)\cdot\vec{\sigma}(2)|\chi_{MA}^{n}\rangle$

$$=\left\langle -\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\middle|\vec{\sigma}(1)\cdot\vec{\sigma}(2)\middle|\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\downarrow\right\rangle$$

$$=\left\langle -\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\middle|\begin{matrix}\sigma_x(1)\sigma_x(2)\\+\sigma_y(1)\sigma_y(2)\\+\sigma_z(1)\sigma_z(2)\end{matrix}\middle|\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\downarrow\right\rangle$$

$$=0+0+0=0.$$

While the detail calculation are shown as below,

$$\left\langle -\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\middle|\sigma_x(1)\sigma_x(2)\middle|\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\downarrow\right\rangle$$

$$=\left\langle -\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\middle|\frac{1}{\sqrt{2}}[(\downarrow\uparrow-\uparrow\downarrow)\downarrow]\right\rangle$$

$$=-\frac{1}{\sqrt{12}}\times(1-1)=0\,.$$

$$\left\langle -\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\middle|\sigma_y(1)\sigma_y(2)\middle|\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\downarrow\right\rangle$$

$$=\left\langle -\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\middle|\frac{1}{\sqrt{2}}[(\downarrow\uparrow-\uparrow\downarrow)\downarrow]\right\rangle$$

$$=-\frac{1}{\sqrt{12}}\times(1-1)=0\,.$$

$$\left\langle -\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\middle|\sigma_z(1)\sigma_z(2)\middle|\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\downarrow\right\rangle$$

$$=\left\langle -\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\middle|\frac{1}{\sqrt{2}}(-\uparrow\downarrow+\downarrow\uparrow)\downarrow\right\rangle=-\frac{1}{\sqrt{12}}\times(1-1)$$

$$=0\,.$$

3) $\langle\chi_{MA}^{n}|\vec{\sigma}(1)\cdot\vec{\sigma}(2)|\chi_{MS}^{n}\rangle$

$$=\left\langle \frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\downarrow\middle|\vec{\sigma}(1)\cdot\vec{\sigma}(2)\middle|-\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]\right\rangle$$

$$= \left\langle \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \left| \begin{matrix} \sigma_x(1)\sigma_x(2) \\ +\sigma_y(1)\sigma_y(2) \\ +\sigma_z(1)\sigma_z(2) \end{matrix} \right| - \frac{1}{\sqrt{6}}[(\uparrow\downarrow + \downarrow\uparrow)\downarrow - 2\downarrow\downarrow\uparrow] \right\rangle$$

$$= 0 + 0 + 0 = 0\,.$$

While the detail calculation are shown as below,

$$\left\langle \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \left| \sigma_x(1)\sigma_x(2) \right| - \frac{1}{\sqrt{6}}[(\uparrow\downarrow + \downarrow\uparrow)\downarrow - 2\downarrow\downarrow\uparrow] \right\rangle$$

$$= \left\langle \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \left| - \frac{1}{\sqrt{6}}[(\downarrow\uparrow + \uparrow\downarrow)\downarrow - 2\uparrow\uparrow\uparrow] \right.\right\rangle$$

$$= -\frac{1}{\sqrt{12}} \times (1 - 1) = 0.$$

$$\left\langle \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \left| \sigma_y(1)\sigma_y(2) \right| - \frac{1}{\sqrt{6}}[(\uparrow\downarrow + \downarrow\uparrow)\downarrow - 2\downarrow\downarrow\uparrow] \right\rangle$$

$$= \left\langle \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \left| - \frac{1}{\sqrt{6}}[(\downarrow\uparrow + \uparrow\downarrow)\downarrow + 2\uparrow\uparrow\uparrow] \right.\right\rangle$$

$$= -\frac{1}{\sqrt{12}} \times (1 - 1) = 0\,.$$

$$\left\langle \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \left| \sigma_z(1)\sigma_z(2) \right| - \frac{1}{\sqrt{6}}[(\uparrow\downarrow + \downarrow\uparrow)\downarrow - 2\downarrow\downarrow\uparrow] \right\rangle$$

$$= \left\langle \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \left| - \frac{1}{\sqrt{6}}[(-\uparrow\downarrow - \downarrow\uparrow)\downarrow - 2\downarrow\downarrow\uparrow] \right.\right\rangle$$

$$= -\frac{1}{\sqrt{12}} \times (1 - 1) = 0\,.$$

4) $$\langle \chi_{MA}^{n} | \vec{\sigma}(1) \cdot \vec{\sigma}(2) | \chi_{MA}^{n} \rangle$$

$$= \left\langle \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \left| \vec{\sigma}(1) \cdot \vec{\sigma}(2) \right| \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \right\rangle$$

$$= \left\langle \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \left| \begin{matrix} \sigma_x(1)\sigma_x(2) \\ +\sigma_y(1)\sigma_y(2) \\ +\sigma_z(1)\sigma_z(2) \end{matrix} \right| \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \right\rangle$$

$$= -1 - 1 - 1 = -3.$$

While the detail calculation are shown as below,

$$\left\langle \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \left| \sigma_x(1)\sigma_x(2) \right| \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \right\rangle$$

$$= \left\langle \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \left| \frac{1}{\sqrt{2}}(\downarrow\uparrow - \uparrow\downarrow)\downarrow \right.\right\rangle$$

$$= \frac{1}{2} \times (-1 - 1) = -1.$$

$$\left\langle \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \middle| \sigma_y(1)\sigma_y(2) \middle| \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \right\rangle$$

$$\left\langle \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \middle| \frac{1}{\sqrt{2}}(i\downarrow(-i)\uparrow - (-i)\uparrow i\downarrow)\downarrow \right\rangle$$

$$= \frac{1}{2} \times (-1-1) = -1.$$

$$\left\langle \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \middle| \sigma_z(1)\sigma_z(2) \middle| \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \right\rangle$$

$$= \left\langle \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow \middle| \frac{1}{\sqrt{2}}(-\uparrow\downarrow + \downarrow\uparrow)\downarrow \right\rangle$$

$$= \frac{1}{2} \times (-1-1) = -1.$$

And then, the final result of $j = 1$ is shown below,

$$\langle n\uparrow |\, \lambda_1(1)\, \lambda_1(2) \vec{\sigma}(1) \cdot \vec{\sigma}(2) | n\uparrow \rangle$$
$$= \tfrac{1}{2} \langle \phi^n_{MS} |\, \lambda_1(1)\, \lambda_1(2) | \phi^n_{MS} \rangle \langle \chi^n_{MS} | \vec{\sigma}(1) \cdot \vec{\sigma}(2) | \chi^n_{MS} \rangle$$
$$+ \tfrac{1}{2} \langle \phi^n_{MS} |\, \lambda_1(1)\, \lambda_1(2) | \phi^n_{MA} \rangle \langle \chi^n_{MS} | \vec{\sigma}(1) \cdot \vec{\sigma}(2) | \chi^n_{MA} \rangle$$
$$+ \tfrac{1}{2} \langle \phi^n_{MA} |\, \lambda_1(1)\, \lambda_1(2) | \phi^n_{MS} \rangle \langle \chi^n_{MA} | \vec{\sigma}(1) \cdot \vec{\sigma}(2) | \chi^n_{MS} \rangle$$
$$+ \tfrac{1}{2} \langle \phi^n_{MA} |\, \lambda_1(1)\, \lambda_1(2) | \phi^n_{MA} \rangle \langle \chi^n_{MA} | \vec{\sigma}(1) \cdot \vec{\sigma}(2) | \chi^n_{MA} \rangle\,.$$
$$= \tfrac{1}{2}\left(\tfrac{1}{3} \times 1\right) + \tfrac{1}{2}(0 \times 0) + \tfrac{1}{2}(0 \times 0) + \tfrac{1}{2}(-1 \times -3) = \tfrac{1}{2}\left(\tfrac{1}{3} + 3\right) = \tfrac{5}{3}$$

Similarly, we can obtain the results of $j = 2$ and 3.

$$\langle n\uparrow |\, \lambda_2(1)\, \lambda_2(2) \vec{\sigma}(1) \cdot \vec{\sigma}(2) | n\uparrow \rangle$$
$$= \tfrac{1}{2} \langle \phi^n_{MS} |\, \lambda_2(1)\, \lambda_2(2) | \phi^n_{MS} \rangle \langle \chi^n_{MS} | \vec{\sigma}(1) \cdot \vec{\sigma}(2) | \chi^n_{MS} \rangle$$
$$+ \tfrac{1}{2} \langle \phi^n_{MS} |\, \lambda_2(1)\, \lambda_2(2) | \phi^n_{MA} \rangle \langle \chi^n_{MS} | \vec{\sigma}(1) \cdot \vec{\sigma}(2) | \chi^n_{MA} \rangle$$
$$+ \tfrac{1}{2} \langle \phi^n_{MA} |\, \lambda_2(1)\, \lambda_2(2) | \phi^n_{MS} \rangle \langle \chi^n_{MA} | \vec{\sigma}(1) \cdot \vec{\sigma}(2) | \chi^n_{MS} \rangle$$
$$+ \tfrac{1}{2} \langle \phi^n_{MA} |\, \lambda_2(1)\, \lambda_2(2) | \phi^n_{MA} \rangle \langle \chi^n_{MA} | \vec{\sigma}(1) \cdot \vec{\sigma}(2) | \chi^n_{MA} \rangle\,.$$
$$= \frac{5}{3}.$$

$$\langle n\uparrow |\, \lambda_3(1)\, \lambda_3(2) \vec{\sigma}(1) \cdot \vec{\sigma}(2) | n\uparrow \rangle$$
$$= \tfrac{1}{2} \langle \phi^n_{MS} |\, \lambda_3(1)\, \lambda_3(2) | \phi^n_{MS} \rangle \langle \chi^n_{MS} | \vec{\sigma}(1) \cdot \vec{\sigma}(2) | \chi^n_{MS} \rangle$$
$$+ \tfrac{1}{2} \langle \phi^n_{MS} |\, \lambda_3(1)\, \lambda_3(2) | \phi^n_{MA} \rangle \langle \chi^n_{MS} | \vec{\sigma}(1) \cdot \vec{\sigma}(2) | \chi^n_{MA} \rangle$$
$$+ \tfrac{1}{2} \langle \phi^n_{MA} |\, \lambda_3(1)\, \lambda_3(2) | \phi^n_{MS} \rangle \langle \chi^n_{MA} | \vec{\sigma}(1) \cdot \vec{\sigma}(2) | \chi^n_{MS} \rangle$$

$$+\frac{1}{2}\langle\phi^n_{MA}|\;\lambda_3(1)\;\lambda_3(2)|\phi^n_{MA}\rangle\langle\chi^n_{MA}|\vec{\sigma}(1)\cdot\vec{\sigma}(2)|\chi^n_{MA}\rangle\;.$$

$$=\frac{5}{3}.$$

Similarly,

$$\langle n\uparrow|\;\lambda_1(1)\;\lambda_1(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|n\uparrow\rangle$$

$$=\frac{1}{2}\langle\phi^n_{MS}|\;\lambda_1(1)\;\lambda_1(3)|\phi^n_{MS}\rangle\langle\chi^n_{MS}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^n_{MS}\rangle$$

$$+\frac{1}{2}\langle\phi^n_{MS}|\;\lambda_1(1)\;\lambda_1(3)|\phi^n_{MA}\rangle\langle\chi^n_{MS}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^n_{MA}\rangle$$

$$+\frac{1}{2}\langle\phi^n_{MA}|\;\lambda_1(1)\;\lambda_1(3)|\phi^n_{MS}\rangle\langle\chi^n_{MA}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^n_{MS}\rangle$$

$$+\frac{1}{2}\langle\phi^n_{MA}|\;\lambda_1(1)\;\lambda_1(3)|\phi^n_{MA}\rangle\langle\chi^n_{MA}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^n_{MA}\rangle\;.$$

$$=\frac{5}{3}.$$

$$\langle n\uparrow|\;\lambda_2(1)\;\lambda_2(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|n\uparrow\rangle$$

$$=\frac{1}{2}\langle\phi^n_{MS}|\;\lambda_2(1)\;\lambda_2(3)|\phi^n_{MS}\rangle\langle\chi^n_{MS}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^n_{MS}\rangle$$

$$+\frac{1}{2}\langle\phi^n_{MS}|\;\lambda_2(1)\;\lambda_2(3)|\phi^n_{MA}\rangle\langle\chi^n_{MS}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^n_{MA}\rangle$$

$$+\frac{1}{2}\langle\phi^n_{MA}|\;\lambda_2(1)\;\lambda_2(3)|\phi^n_{MS}\rangle\langle\chi^n_{MA}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^n_{MS}\rangle$$

$$+\frac{1}{2}\langle\phi^n_{MA}|\;\lambda_2(1)\;\lambda_2(3)|\phi^n_{MA}\rangle\langle\chi^n_{MA}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^n_{MA}\rangle\;.$$

$$=\frac{5}{3}.$$

$$\langle n\uparrow|\;\lambda_3(2)\;\lambda_3(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|n\uparrow\rangle$$

$$=\frac{1}{2}\langle\phi^n_{MS}|\;\lambda_3(2)\;\lambda_3(3)|\phi^n_{MS}\rangle\langle\chi^n_{MS}|\vec{\sigma}(2)\cdot\vec{\sigma}(3)|\chi^n_{MS}\rangle$$

$$+\frac{1}{2}\langle\phi^n_{MS}|\;\lambda_3(2)\;\lambda_3(3)|\phi^n_{MA}\rangle\langle\chi^n_{MS}|\vec{\sigma}(2)\cdot\vec{\sigma}(3)|\chi^n_{MA}\rangle$$

$$+\frac{1}{2}\langle\phi^n_{MA}|\;\lambda_3(2)\;\lambda_3(3)|\phi^n_{MS}\rangle\langle\chi^n_{MA}|\vec{\sigma}(2)\cdot\vec{\sigma}(3)|\chi^n_{MS}\rangle$$

$$+\frac{1}{2}\langle\phi^n_{MA}|\;\lambda_3(2)\;\lambda_3(3)|\phi^n_{MA}\rangle\langle\chi^n_{MA}|\vec{\sigma}(2)\cdot\vec{\sigma}(3)|\chi^n_{MA}\rangle\;.$$

$$=\frac{5}{3}.$$

Then the total pion contribution of this diagram of the neutron is equal to

$$2\times\left\{\left(\frac{5}{3}+\frac{5}{3}+\frac{5}{3}\right)+\left(\frac{5}{3}+\frac{5}{3}+\frac{5}{3}\right)+\left(\frac{5}{3}+\frac{5}{3}+\frac{5}{3}\right)\right\}=30.$$

Next, we consider the kaon contribution, which is$j=4{,}5{,}6{,}7$.

$$\langle n\uparrow|\;\lambda_4(1)\;\lambda_4(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|n\uparrow\rangle+\langle n\uparrow|\;\lambda_5(1)\;\lambda_5(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|n\uparrow\rangle$$

$$+\langle n\uparrow|\;\lambda_6(1)\;\lambda_6(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|n\uparrow\rangle+\langle n\uparrow|\;\lambda_7(1)\;\lambda_7(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|n\uparrow\rangle$$

Similarly, we can get the results,

$$\langle n\uparrow|\,\lambda_4(1)\,\lambda_4(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|n\uparrow\rangle = 0\ ,$$
$$\langle n\uparrow|\,\lambda_5(1)\,\lambda_5(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|n\uparrow\rangle = 0\ ,$$
$$\langle n\uparrow|\,\lambda_6(1)\,\lambda_6(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|n\uparrow\rangle = 0\ ,$$
$$\langle n\uparrow|\,\lambda_7(1)\,\lambda_7(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|n\uparrow\rangle = 0.$$

Also the results of calculation between the quark number 1 and number 3, and the quark number 2 and number 3, we obtain,

$$\langle n\uparrow|\,\lambda_4(1)\,\lambda_4(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|n\uparrow\rangle = 0\ ,$$
$$\langle n\uparrow|\,\lambda_5(1)\,\lambda_5(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|n\uparrow\rangle = 0\ ,$$
$$\langle n\uparrow|\,\lambda_6(1)\,\lambda_6(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|n\uparrow\rangle = 0\ ,$$
$$\langle n\uparrow|\,\lambda_7(1)\,\lambda_7(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|n\uparrow\rangle = 0\ .$$
$$\langle n\uparrow|\,\lambda_4(2)\,\lambda_4(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|n\uparrow\rangle = 0\ ,$$
$$\langle n\uparrow|\,\lambda_5(2)\,\lambda_5(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|n\uparrow\rangle = 0\ ,$$
$$\langle n\uparrow|\,\lambda_6(2)\,\lambda_6(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|n\uparrow\rangle = 0\ ,$$
$$\langle n\uparrow|\,\lambda_7(2)\,\lambda_7(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|n\uparrow\rangle = 0\ .$$

From above results, we found that the kaon contribution for this diagram is equal to zero.

And for the last one, we consider the eta contribution, which is $j = 8$ in the same way.

$$\langle n\uparrow|\,\lambda_8(1)\,\lambda_8(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|n\uparrow\rangle = -\tfrac{1}{3}\ ,$$
$$\langle n\uparrow|\,\lambda_8(1)\,\lambda_8(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|n\uparrow\rangle = -\tfrac{1}{3}\ ,$$
$$\langle n\uparrow|\,\lambda_8(2)\,\lambda_8(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|n\uparrow\rangle = -\tfrac{1}{3}\ ,$$

We sum the results of eta contribution as below,

$$=2\langle n\uparrow|\,\lambda_8(1)\,\lambda_8(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|n\uparrow\rangle$$
$$+2\langle n\uparrow|\,\lambda_8(1)\,\lambda_8(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|n\uparrow\rangle$$
$$+2\langle n\uparrow|\,\lambda_8(2)\,\lambda_8(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|n\uparrow\rangle\ ,$$
$$= 2\left(-\frac{1}{3}-\frac{1}{3}-\frac{1}{3}\right) = -2.$$

Next, we evaluate the matrix element of proton in the same way same neutron.

$$\langle p\uparrow|\,\lambda_1(1)\,\lambda_1(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|p\uparrow\rangle$$
$$= \tfrac{1}{2}\langle\phi^p_{MS}|\,\lambda_1(1)\,\lambda_1(2)|\phi^p_{MS}\rangle\langle\chi^p_{MS}|\vec{\sigma}(1)\cdot\vec{\sigma}(2)|\chi^p_{MS}\rangle$$
$$+\tfrac{1}{2}\langle\phi^p_{MS}|\,\lambda_1(1)\,\lambda_1(2)|\phi^p_{MA}\rangle\langle\chi^p_{MS}|\vec{\sigma}(1)\cdot\vec{\sigma}(2)|\chi^p_{MA}\rangle$$
$$+\tfrac{1}{2}\langle\phi^p_{MA}|\,\lambda_1(1)\,\lambda_1(2)|\phi^p_{MS}\rangle\langle\chi^p_{MA}|\vec{\sigma}(1)\cdot\vec{\sigma}(2)|\chi^p_{MS}\rangle$$
$$+\frac{1}{2}\langle\phi^p_{MA}|\,\lambda_1(1)\,\lambda_1(2)|\phi^p_{MA}\rangle\langle\chi^p_{MA}|\vec{\sigma}(1)\cdot\vec{\sigma}(2)|\chi^p_{MA}\rangle = \frac{5}{3}.$$

Acting on the quark number 1 and 2, in case of $j = 2$,

$$\langle p\uparrow|\,\lambda_2(1)\,\lambda_2(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|p\uparrow\rangle$$
$$=\tfrac{1}{2}\langle\phi^p_{MS}|\,\lambda_2(1)\,\lambda_2(2)|\phi^p_{MS}\rangle\langle\chi^p_{MS}|\vec{\sigma}(1)\cdot\vec{\sigma}(2)|\chi^p_{MS}\rangle$$
$$+\tfrac{1}{2}\langle\phi^p_{MS}|\,\lambda_2(1)\,\lambda_2(2)|\phi^p_{MA}\rangle\langle\chi^p_{MS}|\vec{\sigma}(1)\cdot\vec{\sigma}(2)|\chi^p_{MA}\rangle$$
$$+\tfrac{1}{2}\langle\phi^p_{MA}|\,\lambda_2(1)\,\lambda_2(2)|\phi^p_{MS}\rangle\langle\chi^p_{MA}|\vec{\sigma}(1)\cdot\vec{\sigma}(2)|\chi^p_{MS}\rangle$$
$$+\frac{1}{2}\langle\phi^p_{MA}|\,\lambda_2(1)\,\lambda_2(2)|\phi^p_{MA}\rangle\langle\chi^p_{MA}|\vec{\sigma}(1)\cdot\vec{\sigma}(2)|\chi^p_{MA}\rangle=\frac{5}{3}.$$

Acting on the quark number 1 and 2, in case of $j=3$,

$$\langle p\uparrow|\,\lambda_3(1)\,\lambda_3(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|p\uparrow\rangle$$
$$=\tfrac{1}{2}\langle\phi^p_{MS}|\,\lambda_3(1)\,\lambda_3(2)|\phi^p_{MS}\rangle\langle\chi^p_{MS}|\vec{\sigma}(1)\cdot\vec{\sigma}(2)|\chi^p_{MS}\rangle$$
$$+\tfrac{1}{2}\langle\phi^p_{MS}|\,\lambda_3(1)\,\lambda_3(2)|\phi^p_{MA}\rangle\langle\chi^p_{MS}|\vec{\sigma}(1)\cdot\vec{\sigma}(2)|\chi^p_{MA}\rangle$$
$$+\tfrac{1}{2}\langle\phi^p_{MA}|\,\lambda_3(1)\,\lambda_3(2)|\phi^p_{MS}\rangle\langle\chi^p_{MA}|\vec{\sigma}(1)\cdot\vec{\sigma}(2)|\chi^p_{MS}\rangle$$
$$+\frac{1}{2}\langle\phi^p_{MA}|\,\lambda_3(1)\,\lambda_3(2)|\phi^p_{MA}\rangle\langle\chi^p_{MA}|\vec{\sigma}(1)\cdot\vec{\sigma}(2)|\chi^p_{MA}\rangle=\frac{5}{3}.$$

Acting on the quark number 1 and 3, and $j=1,2,3$

In case of $j=1$,

$$\langle p\uparrow|\,\lambda_j(1)\,\lambda_j(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|p\uparrow\rangle$$
$$=\tfrac{1}{2}\langle\phi^p_{MS}|\,\lambda_1(1)\,\lambda_1(3)|\phi^p_{MS}\rangle\langle\chi^p_{MS}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^p_{MS}\rangle$$
$$+\tfrac{1}{2}\langle\phi^p_{MS}|\,\lambda_1(1)\,\lambda_1(3)|\phi^p_{MA}\rangle\langle\chi^p_{MS}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^p_{MA}\rangle$$
$$+\tfrac{1}{2}\langle\phi^p_{MA}|\,\lambda_1(1)\,\lambda_1(3)|\phi^p_{MS}\rangle\langle\chi^p_{MA}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^p_{MS}\rangle$$
$$+\frac{1}{2}\langle\phi^p_{MA}|\,\lambda_1(1)\,\lambda_1(3)|\phi^p_{MA}\rangle\langle\chi^p_{MA}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^p_{MA}\rangle=\frac{5}{3}.$$

In case of $j=2$,

$$\langle p\uparrow|\,\lambda_j(1)\,\lambda_j(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|p\uparrow\rangle$$
$$=\tfrac{1}{2}\langle\phi^p_{MS}|\,\lambda_2(1)\,\lambda_2(3)|\phi^p_{MS}\rangle\langle\chi^p_{MS}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^p_{MS}\rangle$$
$$+\tfrac{1}{2}\langle\phi^p_{MS}|\,\lambda_2(1)\,\lambda_2(3)|\phi^p_{MA}\rangle\langle\chi^p_{MS}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^p_{MA}\rangle$$
$$+\tfrac{1}{2}\langle\phi^p_{MA}|\,\lambda_2(1)\,\lambda_2(3)|\phi^p_{MS}\rangle\langle\chi^p_{MA}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^p_{MS}\rangle$$
$$+\frac{1}{2}\langle\phi^p_{MA}|\,\lambda_2(1)\,\lambda_2(3)|\phi^p_{MA}\rangle\langle\chi^p_{MA}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^p_{MA}\rangle=\frac{5}{3}.$$

In case of $j=3$,

$$\langle p\uparrow|\,\lambda_j(1)\,\lambda_j(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|p\uparrow\rangle$$
$$=\tfrac{1}{2}\langle\phi^p_{MS}|\,\lambda_3(1)\,\lambda_3(3)|\phi^p_{MS}\rangle\langle\chi^p_{MS}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^p_{MS}\rangle$$

$$+\frac{1}{2}\langle\phi^p_{MS}|\,\lambda_3(1)\,\lambda_3(3)|\phi^p_{MA}\rangle\langle\chi^p_{MS}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^p_{MA}\rangle$$
$$+\frac{1}{2}\langle\phi^p_{MA}|\,\lambda_3(1)\,\lambda_3(3)|\phi^p_{MS}\rangle\langle\chi^p_{MA}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^p_{MS}\rangle$$
$$+\frac{1}{2}\langle\phi^p_{MA}|\,\lambda_3(1)\,\lambda_3(3)|\phi^p_{MA}\rangle\langle\chi^p_{MA}|\vec{\sigma}(1)\cdot\vec{\sigma}(3)|\chi^p_{MA}\rangle=\frac{5}{3}.$$

Next, to evaluate the $\langle p\uparrow|\,\lambda_j(2)\,\lambda_j(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|p\uparrow\rangle$, which the operators are acting on the quark number 2 and the quark number 3, in case of $j=1,2,3$

In case of $j=1$,

$$\langle p\uparrow|\,\lambda_j(2)\,\lambda_j(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|p\uparrow\rangle$$
$$=\frac{1}{2}\langle\phi^p_{MS}|\,\lambda_1(2)\,\lambda_1(3)|\phi^p_{MS}\rangle\langle\chi^p_{MS}|\vec{\sigma}(2)\cdot\vec{\sigma}(3)|\chi^p_{MS}\rangle$$
$$+\frac{1}{2}\langle\phi^p_{MS}|\,\lambda_1(2)\,\lambda_1(3)|\phi^p_{MA}\rangle\langle\chi^p_{MS}|\vec{\sigma}(2)\cdot\vec{\sigma}(3)|\chi^p_{MA}\rangle$$
$$+\frac{1}{2}\langle\phi^p_{MA}|\,\lambda_1(2)\,\lambda_1(3)|\phi^p_{MS}\rangle\langle\chi^p_{MA}|\vec{\sigma}(2)\cdot\vec{\sigma}(3)|\chi^p_{MS}\rangle$$
$$+\frac{1}{2}\langle\phi^p_{MA}|\,\lambda_1(2)\,\lambda_1(3)|\phi^p_{MA}\rangle\langle\chi^p_{MA}|\vec{\sigma}(2)\cdot\vec{\sigma}(3)|\chi^p_{MA}\rangle=\frac{5}{3}.$$

In case of $j=2$,

$$\langle p\uparrow|\,\lambda_j(2)\,\lambda_j(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|p\uparrow\rangle$$
$$=\frac{1}{2}\langle\phi^p_{MS}|\,\lambda_2(2)\,\lambda_2(3)|\phi^p_{MS}\rangle\langle\chi^p_{MS}|\vec{\sigma}(2)\cdot\vec{\sigma}(3)|\chi^p_{MS}\rangle$$
$$+\frac{1}{2}\langle\phi^p_{MS}|\,\lambda_2(2)\,\lambda_2(3)|\phi^p_{MA}\rangle\langle\chi^p_{MS}|\vec{\sigma}(2)\cdot\vec{\sigma}(3)|\chi^p_{MA}\rangle$$
$$+\frac{1}{2}\langle\phi^p_{MA}|\,\lambda_2(2)\,\lambda_2(3)|\phi^p_{MS}\rangle\langle\chi^p_{MA}|\vec{\sigma}(2)\cdot\vec{\sigma}(3)|\chi^p_{MS}\rangle$$
$$+\frac{1}{2}\langle\phi^p_{MA}|\,\lambda_2(2)\,\lambda_2(3)|\phi^p_{MA}\rangle\langle\chi^p_{MA}|\vec{\sigma}(2)\cdot\vec{\sigma}(3)|\chi^p_{MA}\rangle=\frac{5}{3}.$$

In case of $j=3$,

$$\langle p\uparrow|\,\lambda_j(2)\,\lambda_j(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|p\uparrow\rangle$$
$$=\frac{1}{2}\langle\phi^p_{MS}|\,\lambda_3(2)\,\lambda_3(3)|\phi^p_{MS}\rangle\langle\chi^p_{MS}|\vec{\sigma}(2)\cdot\vec{\sigma}(3)|\chi^p_{MS}\rangle$$
$$+\frac{1}{2}\langle\phi^p_{MS}|\,\lambda_3(2)\,\lambda_3(3)|\phi^p_{MA}\rangle\langle\chi^p_{MS}|\vec{\sigma}(2)\cdot\vec{\sigma}(3)|\chi^p_{MA}\rangle$$
$$+\frac{1}{2}\langle\phi^p_{MA}|\,\lambda_3(2)\,\lambda_3(3)|\phi^p_{MS}\rangle\langle\chi^p_{MA}|\vec{\sigma}(2)\cdot\vec{\sigma}(3)|\chi^p_{MS}\rangle$$
$$+\frac{1}{2}\langle\phi^p_{MA}|\,\lambda_3(2)\,\lambda_3(3)|\phi^p_{MA}\rangle\langle\chi^p_{MA}|\vec{\sigma}(2)\cdot\vec{\sigma}(3)|\chi^p_{MA}\rangle=\frac{5}{3}.$$

We summarize the result of Pion contribution as below,

$$2\langle p\uparrow|\,\lambda_j(1)\,\lambda_j(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|p\uparrow\rangle$$
$$+2\langle p\uparrow|\,\lambda_j(1)\,\lambda_j(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|p\uparrow\rangle$$
$$+2\langle p\uparrow|\,\lambda_j(2)\,\lambda_j(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|p\uparrow\rangle$$
$$=2\times\left(\frac{5}{3}+\frac{5}{3}+\frac{5}{3}\right)+2\left(\frac{5}{3}+\frac{5}{3}+\frac{5}{3}\right)+2\left(\frac{5}{3}+\frac{5}{3}+\frac{5}{3}\right)=30$$

Next, to evaluate the $\langle p\uparrow | \lambda_j(1)\,\lambda_j(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|p\uparrow\rangle$, which the operators are acting on the quark number 1 and the quark number 2, in case of Kaon contribution, which are $j = 4,5,6,7$. By using some algebra and calculate straight forwardly, we can get the below results.

$$\langle p\uparrow | \lambda_4(1)\,\lambda_4(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|p\uparrow\rangle = 0\,,$$
$$\langle p\uparrow | \lambda_5(1)\,\lambda_5(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|p\uparrow\rangle = 0\,,$$
$$\langle p\uparrow | \lambda_6(1)\,\lambda_6(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|p\uparrow\rangle = 0\,,$$
$$\langle p\uparrow | \lambda_7(1)\,\lambda_7(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|p\uparrow\rangle = 0\,.$$

And to evaluate the $\langle p\uparrow | \lambda_j(1)\,\lambda_j(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|p\uparrow\rangle$, which the operators are acting on the quark number 1 and the quark number 3, in case of Kaon contribution,

$$\langle p\uparrow | \lambda_4(1)\,\lambda_4(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|p\uparrow\rangle = 0\,,$$
$$\langle p\uparrow | \lambda_5(1)\,\lambda_5(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|p\uparrow\rangle = 0\,,$$
$$\langle p\uparrow | \lambda_6(1)\,\lambda_6(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|p\uparrow\rangle = 0\,,$$
$$\langle p\uparrow | \lambda_7(1)\,\lambda_7(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|p\uparrow\rangle = 0\,.$$

And to evaluate the $\langle p\uparrow | \lambda_j(2)\,\lambda_j(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|p\uparrow\rangle$, in case of Kaon contribution,

$$\langle p\uparrow | \lambda_4(2)\,\lambda_4(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|p\uparrow\rangle = 0\,,$$
$$\langle p\uparrow | \lambda_5(2)\,\lambda_5(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|p\uparrow\rangle = 0\,,$$
$$\langle p\uparrow | \lambda_6(2)\,\lambda_6(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|p\uparrow\rangle = 0\,,$$
$$\langle p\uparrow | \lambda_7(2)\,\lambda_7(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|p\uparrow\rangle = 0\,,$$

We summarize the result of Kaon contribution as below,

$$=2\langle p\uparrow | \lambda_j(1)\,\lambda_j(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|p\uparrow\rangle + 2\langle p\uparrow | \lambda_j(1)\,\lambda_j(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|p\uparrow\rangle$$
$$+2\langle p\uparrow | \lambda_j(2)\,\lambda_j(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|p\uparrow\rangle\,,$$
$$= 0 + 0 + 0 + 0 = 0.$$

Next, to evaluate the $\langle p\uparrow | \lambda_j(I)\,\lambda_j(K)\vec{\sigma}(I)\cdot\vec{\sigma}(K)|p\uparrow\rangle$, in case of Eta contribution, which are $j = 8$. By using some algebra we can get,

$$\langle p\uparrow | \lambda_8(1)\,\lambda_8(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|p\uparrow\rangle = -\frac{1}{3}\,,$$
$$\langle p\uparrow | \lambda_8(1)\,\lambda_8(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|p\uparrow\rangle = -\frac{1}{3}\,,$$
$$\langle p\uparrow | \lambda_8(2)\,\lambda_8(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|p\uparrow\rangle = -\frac{1}{3}\,,$$

We summarize the result of Eta contribution as below,

$$=2\langle p\uparrow|\,\lambda_8(1)\,\lambda_8(2)\vec{\sigma}(1)\cdot\vec{\sigma}(2)|p\uparrow\rangle+2\langle p\uparrow|\,\lambda_8(1)\,\lambda_8(3)\vec{\sigma}(1)\cdot\vec{\sigma}(3)|p\uparrow\rangle$$

$$+2\langle p\uparrow|\,\lambda_8(2)\,\lambda_8(3)\vec{\sigma}(2)\cdot\vec{\sigma}(3)|p\uparrow\rangle\,,$$

$$=2\left(-\frac{1}{3}-\frac{1}{3}-\frac{1}{3}\right)=-2.$$

Appendix D

Chiral invariance in SU (3)

Chiral invariance in SU (3)

We consider the transformation of the wave function by a chiral rotation, $e^{i\gamma_5 \frac{\lambda_i \phi_i}{2F}}$. Then, the quark wave functions become,

$$\psi'(x) = e^{i\gamma_5 \frac{\lambda_i \phi_i}{2F}} \psi \;,\; \bar{\psi}'(x) = \psi^\dagger e^{-i\gamma_5 \frac{\lambda_i \phi_i}{2F}} \gamma^0$$

We consider the interaction term $\bar{\psi}'(x) S(r) \psi'(x)$, when ψ is a quark field, ϕ_i is a meson field, and then we obtain,

$$\begin{aligned}
\bar{\psi}'(x) S(r) \psi'(x) &= \psi^\dagger e^{-i\gamma_5 \frac{\lambda_i \phi_i}{2F}} \gamma^0 S(r) e^{i\gamma_5 \frac{\lambda_i \phi_i}{2F}} \psi \quad , \\
&= \psi^\dagger \gamma^0 e^{i\gamma_5 \frac{\lambda_i \phi_i}{2F}} S(r) e^{i\gamma_5 \frac{\lambda_i \phi_i}{2F}} \psi \;, \\
&= \psi^\dagger \gamma^0 \left(1 + i\gamma_5 \frac{\lambda_i \phi_i}{2F}\right) S(r) \left(1 + i\gamma_5 \frac{\lambda_i \phi_i}{2F}\right) \psi \quad , \\
&= \left(\psi^\dagger \gamma^0 + i\psi^\dagger \gamma^0 \gamma_5 \frac{\lambda_i \phi_i}{2F}\right) \left(S(r)\psi + i\gamma_5 S(r) \frac{\lambda_i \phi_i}{2F} \psi\right) \quad , \\
&= \psi^\dagger \gamma^0 S(r) \psi + \psi^\dagger \gamma^0 i\gamma_5 S(r) \frac{\lambda_i \phi_i}{2F} + i\psi^\dagger \gamma^0 \gamma_5 \frac{\lambda_i \phi_i}{2F} S(r) \psi \\
&\quad + i\psi^\dagger \gamma^0 \gamma_5 \frac{\lambda_i \phi_i}{2F} i\gamma_5 S(r) \frac{\lambda_i \phi_i}{2F} \psi \,. \\
&= \bar{\psi} S(r) \psi + \bar{\psi} i\gamma_5 S(r) \frac{\lambda_i \phi_i}{F} \psi + \mathcal{O}\left(\frac{1}{F}\right)^2 .
\end{aligned}$$

In our approach, the terms $\mathcal{O}\left(\frac{1}{F}\right)^2$ can be neglected. And we restrict our calculation to the linear strong interaction term only. And from the properties of bilinear, the transformation of $\bar{\psi} i\gamma_5 S(r) \frac{\lambda_i \phi_i}{F} \psi$ is pseudoscalar transformation. It implies that ϕ_i must be pseudoscalar quantity.

Appendix E

The calculations of the nucleon form factors and some integrations

The calculations of the nucleon form factors and some integrations

1) The evaluation of $\int d^3 x\, \bar{\psi}_\alpha(x)\, \vec{\gamma}\ \psi_\beta(x) e^{i\vec{q}\cdot\vec{x}}$.

We define the quark space part wave functions of α-state and β-state as below,

$$\bar{\psi}_\alpha(x) = \bar{u}_\alpha(\vec{x}) e^{i\varepsilon_\alpha t}\ \text{, and }\ \psi_\beta(x) = u_\beta(\vec{x}) e^{-i\varepsilon_\beta t}\ .$$

$$\int d^3 x\ \bar{\psi}_\alpha(x)\, \vec{\gamma}\ \psi_\beta(x) e^{i\vec{q}\cdot\vec{x}} = \int d^3 x\ \bar{u}_\alpha(x)\, \vec{\gamma}\ u_\beta(x) e^{i\vec{q}\cdot\vec{x}} e^{i(\varepsilon_\alpha - \varepsilon_\beta)t}$$

Where $u_\alpha(\vec{x})$ and $u_\beta(\vec{x})$ are Dirac spinors which are given by,

$$u_\alpha(\vec{x}) = N_\alpha \begin{pmatrix} g_\alpha(r) \\ i\vec{\sigma}\cdot\hat{r} f_\alpha(r) \end{pmatrix},\ u_\beta(\vec{x}) = N_\beta \begin{pmatrix} g_\beta(r) \\ i\vec{\sigma}\cdot\hat{r} f_\beta(r) \end{pmatrix},\ |\vec{x}| = r$$

We progress step by step, at the first step we will neglect the spin part of the quark wave function, but we will include them at the last calculation.

(i) $$\bar{u}_\alpha = u_\alpha^\dagger \gamma^0 = \left(g_\alpha(r)\, , -i\vec{\sigma}\cdot\hat{r} f_\alpha(r)\right)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$
$$= \left(g_\alpha(r)\, , i\vec{\sigma}\cdot\hat{r} f_\alpha(r)\right).$$

(ii) $$\bar{u}_\alpha(\vec{x})\, \vec{\gamma}\, u_\beta(\vec{x})$$
$$= N_\alpha\left(g_\alpha(r)\, , i\vec{\sigma}\cdot\hat{r} f_\alpha(r)\right)\begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} N_\beta \begin{pmatrix} g_\beta(r) \\ i\vec{\sigma}\cdot\hat{r} f_\beta(r) \end{pmatrix},$$
$$= -i\vec{\sigma}\cdot\hat{r} f_\alpha(r)\, \vec{\sigma}\, g_\beta(r) + g_\alpha(r)\, \vec{\sigma}\ i\vec{\sigma}\cdot\hat{r} f_\beta(r)\ ,$$
$$= i g_\alpha(r) f_\beta(r)\, \vec{\sigma}\, (\vec{\sigma}\cdot\hat{r}) - i(\vec{\sigma}\cdot\hat{r})\, \vec{\sigma}\, f_\alpha(r) g_\beta(r)$$

(iii) by using the identities of σ_i

$$[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k \ \rightarrow \sigma_i\sigma_j - \sigma_j\sigma_i = 2\epsilon_{ijk}\sigma_k$$

$$\{\sigma_i, \sigma_j\} = 2\delta_{ij} \rightarrow \sigma_i\sigma_j + \sigma_j\sigma_i = 2\delta_{ij},$$

and then,

$$\sigma_i\sigma_j = \delta_{ij} + i\epsilon_{ijk}\sigma_k$$

and multiply this equation by A_j we get,

$$\sigma_i\sigma_j A_j = \left[\vec{\sigma}\, (\vec{\sigma}\cdot\vec{A})\right]_i = \left(\delta_{ij} + i\epsilon_{ijk}\sigma_k\right) A_j$$

We re-arrange the index of ϵ_{ijk} to be ϵ_{ikj} in order to get a vector cross product.

$$\sigma_i\sigma_j A_j = A_i - i\epsilon_{ikj}\sigma_k A_j$$

We can rewrite the above relation in term of vector dot product and cross product as,

$$\vec{\sigma}\ (\vec{\sigma}\cdot\vec{A}) = \vec{A} - i(\vec{\sigma}\times\vec{A})$$

Similarly, we can get the below relation,

$$(\vec{\sigma}\cdot\vec{A})\,\vec{\sigma}\ = \vec{A} + i(\vec{\sigma}\times\vec{A})$$

Here, we replace the vector $\vec{A}$ by a vector $\hat{r}$, and then we get the relations,

$$\vec{\sigma}\ \vec{\sigma}\cdot\hat{r} - \vec{\sigma}\cdot\hat{r}\ \vec{\sigma}\ = -2i(\vec{\sigma}\times\hat{r})$$

(i) We can get $\bar{u}_{\alpha}(x)\,\vec{\gamma}\,u_{\beta}(\vec{x})$ in terms of two components, $\hat{r}$ component and $(\vec{\sigma}\times\hat{r})$ components which are perpendicular to each other,

$$\begin{aligned}\bar{u}_{\alpha}(\vec{x})\,\vec{\gamma}\,u_{\beta}(\vec{x}) &= ig_{\alpha}(r)f_{\beta}(r)\,\vec{\sigma}\ \vec{\sigma}\cdot\hat{r} \\ &\quad -i\vec{\sigma}\cdot\hat{r}\,\vec{\sigma}\,f_{\alpha}(r)g_{\beta}(r)\,, \\ &= i\hat{r}\left(g_{\alpha}(r)f_{\beta}(r) - f_{\alpha}(r)g_{\beta}(r)\right) \\ &\quad +(\vec{\sigma}\times\hat{r})\left(g_{\alpha}(r)f_{\beta}(r) + f_{\alpha}(r)g_{\beta}(r)\right)\end{aligned}$$

(ii) After some calculations and select an appropriate direction of the external photon, we find that the $\hat{r}$ component vanishes and only the $(\vec{\sigma}\times\hat{r})$ component survives.

$$\bar{u}_{\alpha}(\vec{x})\,\vec{\gamma}\,u_{\beta}(\vec{x})e^{i\vec{q}\cdot\vec{x}} = (\vec{\sigma}\times\hat{r})\left(g_{\alpha}(r)f_{\beta}(r) + f_{\alpha}(r)g_{\beta}(r)\right)e^{i\vec{q}\cdot\vec{x}}.$$

From the relation between $\hat{r}$ and $\hat{q}$ acting on $e^{i\vec{q}\cdot\vec{x}}$, we get,

$$\hat{r}e^{-i\vec{q}\cdot\vec{x}} = \frac{-i}{r}\frac{\partial e^{iqrcos\theta}}{\partial q} = \hat{q}\ cos\theta\ e^{iqrcos\theta}.$$

(iii) Plug the above relation into the equation, and then we obtain,

$$\begin{aligned}\bar{u}_{\alpha}(\vec{x})\,\vec{\gamma}\,u_{\beta}(\vec{x})e^{i\vec{q}\cdot\vec{x}} &= (\vec{\sigma}\times\hat{q})\left(g_{\alpha}(r)f_{\beta}(r) + f_{\alpha}(r)g_{\beta}(r)\right) \\ &\quad \times cos\theta\ e^{iqrcos\theta}.\end{aligned}$$

If we fix the direction of $\vec{q}$ in the direction $\hat{y}$, or $\vec{q} = (0, q, 0)$ then,

$$(\vec{\sigma}\times\hat{q}) = \hat{x}(-\sigma_z) + \hat{z}(\sigma_x).$$

And in this study, we restrict ourselves to the proton and neutron with spin up, then the second term vanishes, so we obtain,

$$(\vec{\sigma} \times \hat{q}) = \hat{x}(-\sigma_z)\,.$$

(iv) And plug this result into the equation we can get,

$$\bar{u}_\alpha(\vec{x})\,\vec{\gamma}\,u_\beta(\vec{x})e^{i\vec{q}\cdot\vec{x}} = -\sigma_z\hat{x}\big(g_\alpha(r)f_\beta(r)+ f_\alpha(r)g_\beta(r)\,\big)$$
$$\times\, cos\theta\; e^{iqrcos\theta}.$$

(v) Finally, we obtain,

$$\int d^3\,x\;\;\bar{\psi}_\alpha(\vec{x})\,\vec{\gamma}\;\;\psi_\beta(\vec{x})\;e^{i\vec{q}\cdot\vec{x}}$$
$$= N_\alpha N_\beta \int_0^\infty dr \int_0^\pi d\theta\;\;sin\theta\;\int_0^{2\pi} d\phi\;\;(-\sigma_z)\hat{x}\big(g_\alpha(r)f_\beta(r)+ f_\alpha(r)g_\beta(r)\,\big)$$
$$\times\, cos\theta e^{iqrcos\theta} e^{i(\mathcal{E}_\alpha-\mathcal{E}_\beta)t}$$

2) The evaluation of the radial part of $\int d^3\,x\;\bar{\psi}_\alpha(\vec{x})\;\gamma^0\psi_\beta(\vec{x})\;e^{i\vec{q}\cdot\vec{x}}$

We precede our calculation similarly as above methods. And then we obtain,

$$\int d^3\,x\;\;\bar{\psi}_\alpha(x)\,\vec{\gamma}\;\;\psi_\beta(x)e^{i\vec{q}\cdot\vec{x}} = \int d^3\,x\;\;\bar{u}_\alpha(x)\,\vec{\gamma}\;\;u_\beta(x)e^{i\vec{q}\cdot\vec{x}}e^{i(\mathcal{E}_\alpha-\mathcal{E}_\beta)t}.$$

Where $u_\alpha(\vec{x})$ and $u_\beta(\vec{x})$ are Dirac spinors which are given by,

$$u_\alpha(\vec{x}) = N_\alpha\begin{pmatrix} g_\alpha(r) \\ i\vec{\sigma}\cdot\hat{r}f_\alpha(r)\end{pmatrix}, u_\beta(\vec{x}) = N_\beta\begin{pmatrix} g_\beta(r) \\ i\vec{\sigma}\cdot\hat{r}f_\beta(r)\end{pmatrix}, |\vec{x}| = r$$

We progress step by step, at the first step we will neglect the spin part of the quark wave function, but we will include them at the last calculation.

(i)
$$\bar{u}_\alpha = u_\alpha^\dagger\,\gamma^0 = \big(g_\alpha(r)\,,-i\vec{\sigma}\cdot\hat{r}f_\alpha(r)\big)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$
$$= \big(g_\alpha(r)\,,i\vec{\sigma}\cdot\hat{r}f_\alpha(r)\big)$$

$$\bar{u}_\alpha\gamma^0 = u_\alpha^\dagger\,\gamma^0\gamma^0 = \big(g_\alpha(r)\,,-i\vec{\sigma}\cdot\hat{r}f_\alpha(r)\big)$$

(ii)
$$\bar{u}_\alpha(\vec{x})\,\gamma^0\,u_\beta(\vec{x}) = N_\alpha\big(g_\alpha(r)\,,-i\vec{\sigma}\cdot\hat{r}f_\alpha(r)\big)\times N_\beta\begin{pmatrix} g_\beta(r) \\ i\vec{\sigma}\cdot\hat{r}f_\beta(r)\end{pmatrix}$$
$$= N_\alpha N_\beta\big\{g_\alpha(r)g_\beta(r) - i\vec{\sigma}\cdot\hat{r}f_\alpha(r)i\vec{\sigma}\cdot\hat{r}f_\beta(r)\big\}$$

By the use of the identity $(\vec{\sigma}\cdot\hat{r})(\vec{\sigma}\cdot\hat{r}) = \hat{r}\cdot\hat{r} + i\,\vec{\sigma}\cdot(\,\hat{r}\times\,\hat{r}) = 1\,.$

$$= N_\alpha N_\beta\big\{g_\alpha(r)g_\beta(r) + f_\alpha(r)f_\beta(r)\big\}$$

And then $\int d^3x\;\bar{u}_\alpha(\vec{x})\;(Q\,\gamma^0\,)u_\beta(\vec{x})\;e^{i\vec{q}\cdot\vec{x}}$ becomes,

$$= N_\alpha N_\beta\int_0^\infty dr\int_0^\pi d\theta\int_0^{2\pi} d\phi\,\big(f_\alpha(r)f_\beta(r) + g_\alpha(r)g_\beta(r)\big)$$
$$\times\, e^{iqrcos\theta} e^{i(\mathcal{E}_\alpha-\mathcal{E}_\beta)t}$$

Evaluation of the product of spin part wave function I_1

$$I_1 = \int d^3 x_1 \;\bar{u}_0(\vec{x}_1)\, i\gamma^5 S(x_1)\, u_\alpha(\vec{x}_1) e^{i\vec{p}\cdot\vec{x}_1}$$
$$\times \int d^3 x \;\bar{u}_\alpha(\vec{x})\, \vec{\gamma}\; u_\beta(\vec{x}) e^{i\vec{q}\cdot\vec{x}}$$
$$\times \int d^3 x_2 \;\bar{u}_\beta(\vec{x}_2)\, i\gamma^5 S(x_2)\; u_0(\vec{x}_2) e^{-i\vec{p}\cdot\vec{x}_2}$$

We express the spin part of $u_0(\vec{x}_2)$ and $\bar{u}_0(\vec{x}_1)$ as,

$$u_0(\vec{x}_2) = C_0 Y_0^0 |\uparrow\rangle \,, \bar{u}_0(\vec{x}_1) = \langle\uparrow| C_0 Y_0^0$$

And we can also express $u_{\alpha,\beta}(\vec{x})$ with total angular momentum $j = \pm\frac{1}{2}$ as below,

for $j = +\frac{1}{2}$, $\quad |u_\alpha, \uparrow\rangle = A_1 Y_\alpha^0 |\uparrow\rangle + A_2 Y_\alpha^1 |\downarrow\rangle$.

and $j = -\frac{1}{2}$, $\quad |u_\alpha, \downarrow\rangle = A_3 Y_\alpha^{-1} |\uparrow\rangle + A_4 Y_\alpha^0 |\downarrow\rangle$.

Similarly, for the $u_\beta(\vec{x})$,

for $j = +\frac{1}{2}$, $\quad |u_\beta, \uparrow\rangle = B_1 Y_\beta^0 |\uparrow\rangle + B_2 Y_\beta^1 |\downarrow\rangle$,

and $j = -\frac{1}{2}$, $\quad |u_\beta, \downarrow\rangle = B_3 Y_\alpha^{-1} |\uparrow\rangle + B_4 Y_\alpha^0 |\downarrow\rangle$,

Where the Clebsch-Gordan coefficients are $A_1 = \left(l_\alpha 0 \frac{1}{2}\frac{1}{2} \middle| j_\alpha \frac{1}{2}\right)$, $A_2 = \left(l_\alpha 1 \frac{1}{2} - \frac{1}{2} \middle| j_\alpha \frac{1}{2}\right)$, $B_1 = \left(l_\beta 0 \frac{1}{2}\frac{1}{2} \middle| j_\beta \frac{1}{2}\right)$, and $B_2 = \left(l_\beta 1 \frac{1}{2} - \frac{1}{2} \middle| j_\beta \frac{1}{2}\right)$.

We can split the integral of the spin part product I into 4 parts as below,

Part 1 u_α, $j = +\frac{1}{2}$ and u_β , $j = +\frac{1}{2}$

$$\langle\uparrow| C_0 Y_0^0\, \vec{\sigma}\cdot\vec{p}\, (A_1 Y_\alpha^0 |\uparrow\rangle + A_2 Y_\alpha^1 |\downarrow\rangle) \times (\langle\uparrow| A_1 Y_\alpha^{0*} + \langle\downarrow| A_2 Y_\alpha^{1*})$$
$$\times (-\sigma_z \hat{x}) \times \left(B_1 Y_\beta^0 |\uparrow\rangle + B_2 Y_\beta^1 |\downarrow\rangle\right)$$
$$\times \left(\langle\uparrow| B_1 Y_\beta^{0*} + \langle\downarrow| B_2 Y_\beta^{1*}\right) \vec{\sigma}\cdot\vec{p}\, C_0 Y_0^0 |\uparrow\rangle$$

Part 2 u_α, $j = +\frac{1}{2}$ and u_β , $j = -\frac{1}{2}$

$$\langle\uparrow| C_0 Y_0^0\, \vec{\sigma}\cdot\vec{p}\, (A_1 Y_\alpha^0 |\uparrow\rangle + A_2 Y_\alpha^1 |\downarrow\rangle) \times (\langle\uparrow| A_1 Y_\alpha^{0*} + \langle\downarrow| A_2 Y_\alpha^{1*})$$
$$\times (-\sigma_z \hat{x}) \times \left(B_3 Y_\beta^{-1} |\uparrow\rangle + B_4 Y_\beta^0 |\downarrow\rangle\right)$$
$$\times \left(\langle\uparrow| B_3 Y_\beta^{-1*} + \langle\downarrow| B_4 Y_\beta^{0*}\right) \vec{\sigma}\cdot\vec{p}\, C_0 Y_0^0 |\uparrow\rangle$$

Part 3 u_α , $j = -\frac{1}{2}$ and u_β , $j = +\frac{1}{2}$

$$\langle\uparrow| C_0 Y_0^0\, \vec{\sigma}\cdot\vec{p}\, (A_3 Y_\alpha^{-1} |\uparrow\rangle + A_4 Y_\alpha^0 |\downarrow\rangle) \times (\langle\uparrow| A_3 Y_\alpha^{-1*} + \langle\downarrow| A_4 Y_\alpha^{0*})$$
$$\times (-\sigma_z \hat{x}) \times \left(B_1 Y_\beta^0 |\uparrow\rangle + B_2 Y_\beta^1 |\downarrow\rangle\right)$$
$$\times \left(\langle\uparrow| B_1 Y_\beta^{0*} + \langle\downarrow| B_2 Y_\beta^{1*}\right) \vec{\sigma}\cdot\vec{p}\, C_0 Y_0^0 |\uparrow\rangle$$

Part 4 u_α , $j = -\frac{1}{2}$ and u_β , $j = -\frac{1}{2}$

$$\langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}\ (A_3Y_\alpha^{-1}|\uparrow\rangle + A_4Y_\alpha^0|\downarrow\rangle) \times (\langle\uparrow|A_3Y_\alpha^{-1*} + \langle\downarrow|A_4Y_\alpha^{0*})$$
$$\times(-\sigma_z\hat{x}) \times \left(B_3Y_\beta^{-1}|\uparrow\rangle + B_4Y_\beta^0|\downarrow\rangle\right)$$
$$\times\left(\langle\uparrow|B_3Y_\beta^{-1*} + \langle\downarrow|B_4Y_\beta^0\right)\vec{\sigma}\cdot\vec{p}C_0Y_0^0|\uparrow\rangle$$

We sum all the parts together, and after using below properties of spherical harmonic function $Y_\alpha^{m_\alpha}$

(i) $\int_0^{2\pi} Y_{\alpha,\beta}^1\, d\phi = \int_0^{2\pi} Y_{\alpha,\beta}^{-1}\, d\phi = 0$,

(ii) $A_2Y_\alpha^{1*}B_2Y_\beta^1 = A_2Y_\alpha^1B_2Y_\beta^{1*}$,

(iii)$\langle\downarrow|\uparrow\rangle = \langle\uparrow|\downarrow\rangle = 0$.

Part 1:

$$= \langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}\ (A_1Y_\alpha^0|\uparrow\rangle + A_2Y_\alpha^1|\downarrow\rangle) \times (\langle\uparrow|A_1Y_\alpha^{0*} + \langle\downarrow|A_2Y_\alpha^{1*}) \times$$
$$(-\sigma_z\hat{x}) \times \left(B_1Y_\beta^0|\uparrow\rangle + B_2Y_\beta^1|\downarrow\rangle\right) \times \left(\langle\uparrow|B_1Y_\beta^{0*} + \langle\downarrow|B_2Y_\beta^{1*}\right)\vec{\sigma}\cdot$$
$$\vec{p}C_0Y_0^0|\uparrow\rangle$$
$$= \hat{x}\langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}\ (A_1Y_\alpha^0|\uparrow\rangle) \times \left(-A_1Y_\alpha^{0*}B_1Y_\beta^0 + A_2Y_\alpha^{1*}B_2Y_\beta^1\right)$$
$$\times\left(\langle\uparrow|B_1Y_\beta^{0*}\right)\vec{\sigma}\cdot\vec{p}C_0Y_0^0|\uparrow\rangle .$$
$$= \hat{x}\langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}\ A_1Y_\alpha^0 \times \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\left(-A_1Y_\alpha^{0*}B_1Y_\beta^0 + A_2Y_\alpha^{1*}B_2Y_\beta^1\right)$$

Part 2 u_α, $j = +\frac{1}{2}$ and u_β , $j = -\frac{1}{2}$

$$\langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}\ (A_1Y_\alpha^0|\uparrow\rangle + A_2Y_\alpha^1|\downarrow\rangle) \times (\langle\uparrow|A_1Y_\alpha^{0*} + \langle\downarrow|A_2Y_\alpha^{1*})$$
$$\times(-\sigma_z\hat{x}) \times \left(B_3Y_\beta^{-1}|\uparrow\rangle + B_4Y_\beta^0|\downarrow\rangle\right)$$
$$\times\left(\langle\uparrow|B_3Y_\beta^{-1*} + \langle\downarrow|B_4Y_\beta^{0*}\right)\vec{\sigma}\cdot\vec{p}C_0Y_0^0|\uparrow\rangle = 0$$

Part 3 u_α , $j = -\frac{1}{2}$ and u_β , $j = +\frac{1}{2}$

$$\langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}\ (A_3Y_\alpha^{-1}|\uparrow\rangle + A_4Y_\alpha^0|\downarrow\rangle) \times (\langle\uparrow|A_3Y_\alpha^{-1*} + \langle\downarrow|A_4Y_\alpha^{0*})$$
$$\times(-\sigma_z\hat{x}) \times \left(B_1Y_\beta^0|\uparrow\rangle + B_2Y_\beta^1|\downarrow\rangle\right)$$
$$\times\left(\langle\uparrow|B_1Y_\beta^{0*} + \langle\downarrow|B_2Y_\beta^{1*}\right)\vec{\sigma}\cdot\vec{p}C_0Y_0^0|\uparrow\rangle = 0$$

Part 4 u_α , $j = -\frac{1}{2}$ and u_β , $j = -\frac{1}{2}$

$$\langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}\ (A_3Y_\alpha^{-1}|\uparrow\rangle + A_4Y_\alpha^0|\downarrow\rangle) \times (\langle\uparrow|A_3Y_\alpha^{-1*} + \langle\downarrow|A_4Y_\alpha^{0*})$$
$$\times(-\sigma_z\hat{x}) \times \left(B_3Y_\beta^{-1}|\uparrow\rangle + B_4Y_\beta^0|\downarrow\rangle\right)$$
$$\times\left(\langle\uparrow|B_3Y_\beta^{-1*} + \langle\downarrow|B_4Y_\beta^0\right)\vec{\sigma}$$
$$\cdot\,\vec{p}C_0Y_0^0\Big|\uparrow\rangle$$
$$= \hat{x}\langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}\ A_1Y_\alpha^0\begin{pmatrix} -A_3Y_\alpha^{-1*}B_3Y_\beta^{-1} \\ +A_4Y_\alpha^{0*}B_4Y_\beta^0 \end{pmatrix}$$
$$\times\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\left(B_4Y_\beta^0\right)\vec{\sigma}\cdot\vec{p}C_0Y_0^0|\uparrow\rangle.$$

And then,

$$I_1 = \hat{x}\langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}\ A_1Y_\alpha^0 \times \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\left(-A_1Y_\alpha^{0*}B_1Y_\beta^0 + A_2Y_\alpha^{1*}B_2Y_\beta^1\right)$$
$$+\ \hat{x}\langle\uparrow|C_0Y_0^0\ \vec{\sigma}$$
$$\cdot\,\vec{p}\ A_1Y_\alpha^0\left(-A_3Y_\alpha^{-1*}B_3Y_\beta^{-1} + A_4Y_\alpha^{0*}B_4Y_\beta^0\right)$$
$$\times\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\left(B_4Y_\beta^0\right)\vec{\sigma}\cdot\vec{p}C_0Y_0^0|\uparrow\rangle.$$

From the Clebsch-Gordon coefficients relation, we get,

$$-A_1Y_\alpha^{0*}B_1Y_\beta^0 + A_2Y_\alpha^{1*}B_2Y_\beta^1 = A_3Y_\alpha^{-1*}B_3Y_\beta^{-1} - A_4Y_\alpha^{0*}B_4Y_\beta^0$$

Finally,

$$I_1 = \hat{x}\langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}\ A_1Y_\alpha^0\left(-A_1Y_\alpha^{0*}B_1Y_\beta^0 +\right.$$
$$\left.A_2Y_\alpha^{1*}B_2Y_\beta^1\right)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\left(B_4Y_\beta^0\right)\vec{\sigma}\cdot\vec{p}C_0Y_0^0|\uparrow\rangle.$$

Evaluation of the product of spin part wave function I_2

$$I_2 = \int d^3x_1\ \bar{u}_0(\vec{x}_1)\ i\gamma^5 S(x_1)\ u_\alpha(\vec{x}_1)e^{i\vec{p}\cdot\vec{x}_1}$$
$$\times\int d^3x\ \bar{u}_\alpha(\vec{x})\,\gamma_0\ u_\beta(\vec{x})e^{i\vec{q}\cdot\vec{x}}$$
$$\times\int d^3x_2\ \bar{u}_\beta(\vec{x}_2)\ i\gamma^5 S(x_2)\ u_0(\vec{x}_2)e^{-i\vec{p}\cdot\vec{x}_2}$$

We can split the integral of the spin part product I_2 into 4 parts similar to I_1 as below,

Part 1 u_α, $j = +\frac{1}{2}$ and u_β, $j = +\frac{1}{2}$

$$\langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}\ (A_1Y_\alpha^0|\uparrow\rangle + A_2Y_\alpha^1|\downarrow\rangle) \times (\langle\uparrow|A_1Y_\alpha^{0*} + \langle\downarrow|A_2Y_\alpha^{1*})$$
$$\times\left(B_1Y_\beta^0|\uparrow\rangle + B_2Y_\beta^1|\downarrow\rangle\right) \times \left(\langle\uparrow|B_1Y_\beta^{0*} + \langle\downarrow|B_2Y_\beta^{1*}\right)\vec{\sigma}\cdot\vec{p}C_0Y_0^0|\uparrow\rangle$$
$$= \langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}\ (A_1Y_\alpha^0|\uparrow\rangle) \times \left(A_1Y_\alpha^{0*}B_1Y_\beta^0 + A_2Y_\alpha^{1*}B_2Y_\beta^1\right)\left(\langle\uparrow|B_1Y_\beta^{0*}\right)\vec{\sigma}\cdot\vec{p}C_0Y_0^0|\uparrow\rangle$$
$$= \langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}A_1Y_\alpha^0\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\left(A_1Y_\alpha^{0*}B_1Y_\beta^0 + A_2Y_\alpha^{1*}B_2Y_\beta^1\right)\left(B_1Y_\beta^{0*}\right)\vec{\sigma}\cdot\vec{p}C_0Y_0^0|\uparrow\rangle$$

Part 2 $u_\alpha,\ j=+\frac{1}{2}$ and $u_\beta\ ,\ j=-\frac{1}{2}$

$$\langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}\ (A_1Y_\alpha^0|\uparrow\rangle+A_2Y_\alpha^1|\downarrow\rangle)\times(\langle\uparrow|A_1Y_\alpha^{0*}+\langle\downarrow|A_2Y_\alpha^{1*})$$
$$\times\left(B_3Y_\beta^{-1}|\uparrow\rangle+B_4Y_\beta^0|\downarrow\rangle\right)$$
$$\times\left(\langle\uparrow|B_3Y_\beta^{-1*}+\langle\downarrow|B_4Y_\beta^{0*}\right)\vec{\sigma}\cdot\vec{p}C_0Y_0^0|\uparrow\rangle=0.$$

Part 3 $u_\alpha\ ,\ j=-\frac{1}{2}$ and $u_\beta\ ,\ j=+\frac{1}{2}$

$$\langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}\ (A_3Y_\alpha^{-1}|\uparrow\rangle+A_4Y_\alpha^0|\downarrow\rangle)\times(\langle\uparrow|A_3Y_\alpha^{-1*}+\langle\downarrow|A_4Y_\alpha^{0*})$$
$$\times\left(B_1Y_\beta^0|\uparrow\rangle+B_2Y_\beta^1|\downarrow\rangle\right)$$
$$\times\left(\langle\uparrow|B_1Y_\beta^{0*}+\langle\downarrow|B_2Y_\beta^{1*}\right)\vec{\sigma}\cdot\vec{p}C_0Y_0^0|\uparrow\rangle=0.$$

Part 4 $u_\alpha\ ,\ j=-\frac{1}{2}$ and $u_\beta\ ,\ j=-\frac{1}{2}$

$$\langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}\ (A_3Y_\alpha^{-1}|\uparrow\rangle+A_4Y_\alpha^0|\downarrow\rangle)\times(\langle\uparrow|A_3Y_\alpha^{-1*}+\langle\downarrow|A_4Y_\alpha^{0*})$$
$$\times\left(B_3Y_\beta^{-1}|\uparrow\rangle+B_4Y_\beta^0|\downarrow\rangle\right)\times\left(\langle\uparrow|B_3Y_\beta^{-1*}+\langle\downarrow|B_4Y_\beta^0\right)\vec{\sigma}\cdot\vec{p}C_0Y_0^0|\uparrow\rangle$$
$$=\langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}\ A_4Y_\alpha^0|\downarrow\rangle\left(A_3Y_\alpha^{-1*}B_3Y_\beta^{-1}+A_4Y_\alpha^{0*}B_4Y_\beta^0\right)\langle\downarrow|B_4Y_\beta^0\vec{\sigma}\cdot\vec{p}C_0Y_0^0|\uparrow\rangle$$
$$=\langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}\ A_4Y_\alpha^0\left(A_3Y_\alpha^{-1*}B_3Y_\beta^{-1}+A_4Y_\alpha^{0*}B_4Y_\beta^0\right)\begin{pmatrix}0&0\\0&1\end{pmatrix}\left(B_4Y_\beta^0\vec{\sigma}\cdot\vec{p}C_0Y_0^0|\uparrow\rangle\right)$$

From the Clebsch-Gordon coefficients relation, we get,

$$A_1Y_\alpha^{0*}B_1Y_\beta^0+A_2Y_\alpha^{1*}B_2Y_\beta^1=A_3Y_\alpha^{-1*}B_3Y_\beta^{-1}+A_4Y_\alpha^{0*}B_4Y_\beta^0$$

Finally,

$$I_2=\langle\uparrow|C_0Y_0^0\ \vec{\sigma}\cdot\vec{p}A_1Y_\alpha^0\left(A_1Y_\alpha^{0*}B_1Y_\beta^0+A_2Y_\alpha^{1*}B_2Y_\beta^1\right)$$
$$\times\begin{pmatrix}1&0\\0&1\end{pmatrix}\left(B_1Y_\beta^{0*}\right)\vec{\sigma}\cdot\vec{p}C_0Y_0^0|\uparrow\rangle$$

3) The evaluation of $\int d^3x_1\,\bar{u}_0(\vec{x}_1)i\gamma^5S(x_1)u_\alpha(\vec{x}_1)e^{i\vec{p}\cdot\vec{x}_1}$

$$u_0(\vec{x}_1)=N_0\begin{pmatrix}g_0(r)\\i\vec{\sigma}\cdot\hat{r}f_0(r)\end{pmatrix}C_0Y_{0,0}|\uparrow\rangle,$$

$$u_\alpha(\vec{x}_1)=N_\alpha\begin{pmatrix}g_\alpha(r)\\i\vec{\sigma}\cdot\hat{r}f_\alpha(r)\end{pmatrix}A_1Y_{\alpha,0}|\uparrow\rangle\,,$$

and $|\vec{x}_1|=r$.

We progress step by step,

(i) $$\bar{u}_0=u_0^\dagger\gamma^0=\left(g_0(r)\,,-i\vec{\sigma}\cdot\hat{r}f_0(r)\right)\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$
$$=\left(g_0(r)\,,i\vec{\sigma}\cdot\hat{r}f_0(r)\right)\langle\uparrow|\left(C_0Y_{0,0}\right)^*$$

(ii) $$\bar{u}_0(\vec{x}_1)i\gamma^5S(x_1)u_\alpha(\vec{x}_1)$$
$$=iN_0\left(g_0(r)\,,i\vec{\sigma}\cdot\hat{r}f_0(r)\right)\begin{pmatrix}0&1\\1&0\end{pmatrix}N_\alpha\begin{pmatrix}g_\alpha(r)\\i\vec{\sigma}\cdot\hat{r}f_\alpha(r)\end{pmatrix}C_0Y_{0,0}A_1Y_{\alpha,0}S(r),$$
$$=iN_0N_\alpha[i\vec{\sigma}\cdot\hat{r}\,f_0(r)\,g_\alpha(r)+\ i\vec{\sigma}\cdot\hat{r}\,g_0(r)\,f_\alpha(r)]\,C_0Y_{0,0}A_1Y_{\alpha,0}$$

(iii) $$\vec{\sigma}\cdot\hat{r}\,e^{i\vec{p}\cdot\vec{x}_1}=-i\frac{\vec{\sigma}\cdot\hat{p}}{r}\frac{\partial e^{iprcos\theta}}{\partial p}=\vec{\sigma}\cdot\hat{p}\,cos\theta\,e^{iprcos\theta}$$

And then, the result $\int d^3x_1\,\bar{u}_0(\vec{x}_1)i\gamma^5 S(x_1)u_\alpha(\vec{x}_1)e^{i\vec{p}\cdot\vec{x}_1}$

$$= -N_0N_\alpha\vec{\sigma}\cdot\hat{p}\int_0^\infty dr\int_0^\pi d\theta\int_0^{2\pi} d\phi\,(C_0Y_{0,0})(A_1Y_{\alpha,0})sin\theta cos\theta e^{iprcos\theta}$$
$$\times r^2S(r)\big(f_0(r)g_\alpha(r)+(f_\alpha(r)g_0(r)\big)$$

4) The evaluation of $\int d^3x_2\,\bar{u}_\beta(\vec{x}_2)i\gamma^5 S(x_2)u_0(\vec{x}_2)e^{-i\vec{p}\cdot\vec{x}_2}$

$$u_0(\vec{x}_2) = N_0\begin{pmatrix} g_0(r) \\ i\vec{\sigma}\cdot\hat{r}f_0(r)\end{pmatrix}C_0Y_{0,0}|\uparrow\rangle,$$

$$u_\beta(\vec{x}_2) = N_\beta\begin{pmatrix} g_\beta(r) \\ i\vec{\sigma}\cdot\hat{r}f_\beta(r)\end{pmatrix}B_1Y_{\beta,0}\,|\uparrow\rangle,$$

and $|\vec{x}_2| = r$.

We progress step by step,

(i) $$\bar{u}_\beta = u_\beta^\dagger\,\gamma^0 = \big(g_\beta(r)\,,-i\vec{\sigma}\cdot\hat{r}f_\beta(r)\big)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\langle\uparrow|\big(B_1Y_{\beta,0}\big)^*$$
$$= \big(g_\beta(r)\,,i\vec{\sigma}\cdot\hat{r}f_\beta(r)\big)\langle\uparrow|\big(B_1Y_{\beta,0}\big)^*$$

(ii) $$\bar{u}_\beta(\vec{x}_2)i\gamma^5 S(x_2)u_0(\vec{x}_2)$$
$$= iN_\beta\big(g_\beta(r)\,,i\vec{\sigma}\cdot\hat{r}f_\beta(r)\big)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}N_0\begin{pmatrix} g_0(r) \\ i\vec{\sigma}\cdot\hat{r}f_0(r)\end{pmatrix}S(r)C_0Y_{0,0}\big(B_1Y_{\beta,0}\big)^*$$
$$= iN_0N_\beta\big[i\vec{\sigma}\cdot\hat{r}\,f_0(r)\,g_\beta(r)+i\vec{\sigma}\cdot\hat{r}\,g_0(r)\,f_\beta(r)\big]\,S(r)C_0Y_{0,0}\big(B_1Y_{\beta,0}\big)^*$$
$$= -N_0N_\beta\vec{\sigma}\cdot\hat{r}\big[f_0(r)\,g_\beta(r)+\,g_0(r)\,f_\beta(r)\big]$$

(iii) $$\vec{\sigma}\cdot\hat{r}\,e^{-i\vec{p}\cdot\vec{x}_2} = i\frac{\vec{\sigma}\cdot\hat{p}}{r}\frac{\partial e^{-i\vec{p}\cdot\vec{x}_2}}{\partial p}$$
$$= i\frac{\vec{\sigma}\cdot\hat{p}}{r}\frac{\partial e^{-iprcos\theta}}{\partial p}$$
$$= \vec{\sigma}\cdot\hat{p}\;\,cos\theta\;e^{-iprcos\theta}$$

5) The evaluation of $\int d^3x_2\,\bar{u}_\beta(\vec{x}_2)i\gamma^5 S(x_2)u_0(\vec{x}_2)e^{-i\vec{p}\cdot\vec{x}_2}$

$$= -N_0N_\beta\vec{\sigma}\cdot\hat{p}\int_0^\infty dr\int_0^\pi d\theta\int_0^{2\pi} d\phi\,(C_0Y_{0,0})(B_1Y_{\beta,0})sin\theta cos\theta e^{iprcos\theta}$$
$$\times r^2S(r)\big(f_0(r)g_\beta(r)+(f_\beta(r)g_0(r)\big)$$

The evaluation of $G_E^N(Q^2)|_{\alpha,MC}$:

$$G_E^N(Q^2)|_{\alpha,MC}$$

$$= 4\langle\phi_0| -\frac{1}{2}\int \delta(t) d^4x d^4x_1 d^4x_2 e^{-iq\cdot x} : \left(-\bar{\psi}(x_1) i\gamma^5 \frac{\lambda_k}{F} \Phi_k(x_1) S(x_1) \psi_\alpha(x_1)\right)$$

$$\times \left(-\bar{\psi}_\alpha(x_2) i\gamma^5 \frac{\lambda_l}{F} \Phi_l(x_2) S(x_2) \psi(x_2)\right) \times \left(f_{3ij} + \frac{f_{8ij}}{\sqrt{3}}\right) \Phi_i(x) \frac{\partial \Phi_j(x)}{\partial t} : |\phi_0\rangle$$

$$= -\frac{2}{F^2}\langle\phi_0| \int \delta(t) d^4x d^4x_1 d^4x_2 e^{-iq\cdot x}\, \bar{u}_0(\vec{x}_1) e^{i\mathcal{E}_0 t_1} i\gamma^5 \lambda_k S(x_1) u_\alpha(\vec{x}_1) e^{-i\mathcal{E}_\alpha t_1}$$

$$\times \theta(t_1 - t_2)\, \bar{u}_\alpha(\vec{x}_2) e^{i\mathcal{E}_\alpha t_2} i\gamma^5 \lambda_l S(x_2) u_0(\vec{x}_2) e^{-i\mathcal{E}_0 t_2} \times \left(f_{3ij} + \frac{f_{8ij}}{\sqrt{3}}\right)$$

$$\times \delta_{ki} \int \frac{d^4p_1}{(2\pi)^4 i} \frac{e^{-ip_1(x_1 - x)}}{M_\Phi^2 - p_1^2 - i\varepsilon} \times \delta_{lj} \frac{\partial}{\partial t}\left(\int \frac{d^4p_2}{(2\pi)^4 i} \frac{e^{-ip_2(x - x_2)}}{M_\Phi^2 - p_2^2 - i\varepsilon}\right) |\phi_0\rangle$$

Because we restrict the calculation to nucleons with spin up, then we obtain,

$$= -\frac{2i}{(2\pi)^8 F^2}\langle N\uparrow| \int d^4x_1 d^4x_2 d^4p_1 d^4p_2 d^3x\; e^{i\vec{q}\cdot\vec{x}} e^{-i(\mathcal{E}_\alpha - \mathcal{E}_0)t_1} e^{i(\mathcal{E}_\alpha - \mathcal{E}_0)t_2}$$

$$\times \theta(t_1 - t_2) \times \int \delta(t) dt\, e^{-ip_1^0(t_1 - t)} e^{-ip_2^0(t - t_2)} e^{-iq_0 t}$$

$$\times [\bar{u}_0(\vec{x}_1) i\gamma^5 \lambda_i S(x_1) u_\alpha(\vec{x}_1)] \times [\bar{u}_\alpha(\vec{x}_2) i\gamma^5 \lambda_j S(x_2) u_0(\vec{x}_2)]$$

$$\times \left(f_{3ij} + \frac{f_{8ij}}{\sqrt{3}}\right) \times \frac{e^{-i\vec{p}_1(\vec{x}_1 - \vec{x})}}{M_\Phi^2 - p_1^2 - i\varepsilon}\, p_2^0\, \frac{e^{-i\vec{p}_2(\vec{x} - \vec{x}_2)}}{M_\Phi^2 - p_2^2 - i\varepsilon} |N\uparrow\rangle.$$

When we calculate the terms concerning with t only, we can get the below result,

$$\int \delta(t) dt\, e^{-ip_1^0(t_1 - t)} e^{-ip_2^0(t - t_2)} e^{-iq_0 t} = e^{-ip_1^0 t_1} e^{-ip_2^0 t_2}$$

Plug the result back into the equation, and then we get,

$$G_E^N(Q^2)|_{\alpha,MC}$$

$$= -\frac{2i}{(2\pi)^8 F^2}\langle N\uparrow| \int d^3x_1 d^4x_2 d^4p_1 d^4p_2 \times \int d^3x\, e^{i(\vec{q} - \vec{p}_1 + \vec{p}_2)\cdot\vec{x}}$$

$$\times \int dt_1\; e^{-i(p_1^0 + \Delta\mathcal{E}_\alpha)t_1} \theta(t_1 - t_2) \times e^{i(p_2^0 + \Delta\mathcal{E}_\alpha)t_2}$$

$$\times [\bar{u}_0(\vec{x}_1) i\gamma^5 \lambda_i S(x_1) u_\alpha(\vec{x}_1)] \times [\bar{u}_\alpha(\vec{x}_2) i\gamma^5 \lambda_j S(x_2) u_0(\vec{x}_2)]$$

$$\times \left(f_{3ij} + \frac{f_{8ij}}{\sqrt{3}}\right) \times \frac{e^{i\vec{p}_1\cdot\vec{x}_1}}{M_\Phi^2 - p_1^2 - i\varepsilon}\, p_2^0\, \frac{e^{-i\vec{p}_2\cdot\vec{x}_2}}{M_\Phi^2 - p_2^2 - i\varepsilon} |N\uparrow\rangle.$$

Where $\Delta\mathcal{E}_\alpha = \mathcal{E}_\alpha - \mathcal{E}_0$. And make use of the definition of $\theta(t_1 - t_2)$, we get

$$\int dt_1\; e^{-i(p_1^0 + \Delta\mathcal{E}_\alpha)t_1} \theta(t_1 - t_2) = \lim_{\eta \to 0} \frac{-i}{(p_1^0 + \Delta\mathcal{E}_\alpha) - i\eta} e^{-i(p_1^0 + \Delta\mathcal{E}_\alpha)t_2}.$$

And the terms concerning with t_2 become,

$$\int dt_2\ e^{-i(p_1^0+\Delta\mathcal{E}_\alpha)t_2}\ \times e^{i(p_2^0+\Delta\mathcal{E}_\alpha)t_2} = \int dt_2\ e^{i(p_2^0-p_1^0)t_2}$$
$$= 2\pi\delta(p_2^0 - p_1^0)$$

This makes $p_2^0 = p_1^0$. And the terms concerning with t become

$$\int d^3x\, e^{i(\vec{q}-\vec{p}_1+\vec{p}_2)\cdot\vec{x}} = (2\pi)^3\delta(\vec{q} - \vec{p}_1 + \vec{p}_2).$$

This makes $\vec{p}_1 = \vec{q} + \vec{p}_2$.

We define the new $\vec{p} \equiv \vec{p}_2$ and $p_0 \equiv p_2^0 = p_1^0$.And then,

$$G_E^N(Q^2)|_{\alpha,MC}$$
$$= -\frac{2}{(2\pi)^4F^2}\langle N\uparrow|\int d^3p \times \int d^3x_1\,\bar{u}_0(\vec{x}_1)i\gamma^5 S(x_1)u_\alpha(\vec{x}_1)e^{i(\vec{p}+\vec{q})\cdot\vec{x}_1}$$
$$\times \int d^3x_2\bar{u}_\alpha(\vec{x}_2)i\gamma^5 S(x_2)u_0(\vec{x}_2)\,e^{-i\vec{p}\cdot\vec{x}_2} \times \left(f_{3ij} + \frac{f_{8ij}}{\sqrt{3}}\right)\lambda_i\lambda_j$$
$$\times \int dp_0 \frac{1}{M_\Phi^2 + (\vec{p}+\vec{q})^2 - p_0^2 - i\varepsilon}\,\frac{p_0}{(p_0 + \Delta\mathcal{E}_\alpha) - i\eta}\,\frac{1}{M_\Phi^2 + \vec{p}^2 - p_0^2 - i\varepsilon}\,|N\uparrow\rangle.$$

We evaluate the below term by using the Residue theorem,

$$\int_{-\infty}^{\infty} dp_0 \frac{1}{M_\Phi^2 + (\vec{p}+\vec{q})^2 - p_0^2 - i\varepsilon}\,\frac{p_0}{(p_0 + \Delta\mathcal{E}_\alpha) - i\eta}\,\frac{1}{M_\Phi^2 + \vec{p}^2 - p_0^2 - i\varepsilon}$$
$$= -2\pi i \sum_{z_0} Res(f[z_0]).$$

We expand the left hand side of the above equation, then we get,

$$\int dp_0 \frac{1}{(p_0 - \omega_\Phi(\vec{p}+\vec{q}) + i\varepsilon)}\,\frac{1}{(p_0 + \omega_\Phi(\vec{p}+\vec{q}) - i\varepsilon)}$$
$$\times \frac{p_0}{(p_0 + \Delta\mathcal{E}_\alpha) - i\eta}\,\frac{1}{(p_0 - \omega_\Phi(\vec{p}) + i\varepsilon)}\,\frac{1}{(p_0 + \omega_\Phi(\vec{p}) - i\varepsilon)}.$$

Where $\omega_\Phi(\vec{p}+\vec{q}) \equiv \sqrt{M_\Phi^2 + (\vec{p}+\vec{q})^2}$, and $\omega_\Phi(\vec{p}) \equiv \sqrt{M_\Phi^2 + \vec{p}^2}$.

There are 5 poles, three poles in the upper plane and two poles in the lower plane. In our calculation, we select the lower plane for simplicity.

For $z_0 = \omega_\Phi(\vec{p}+\vec{q}) - i\varepsilon$,

$$Res(f\,[z_0])$$
$$= \frac{1}{2\omega_\Phi(\vec{p}+\vec{q})}\,\frac{\omega_\Phi(\vec{p}+\vec{q})}{(\omega_\Phi(\vec{p}+\vec{q}) + \Delta\mathcal{E}_\alpha)}\,\frac{1}{(\omega_\Phi(\vec{p}+\vec{q}) - \omega_\Phi(\vec{p}))}\,\frac{1}{(\omega_\Phi(\vec{p}+\vec{q}) - \omega_\Phi(\vec{p}))}$$
$$= \frac{1}{2}\,\frac{1}{(\omega_\Phi(\vec{p}+\vec{q}) + \Delta\mathcal{E}_\alpha)}\,\frac{1}{\left(\omega_\Phi(\vec{p}+\vec{q}) - \omega_\Phi(\vec{p})\right)}\,\frac{1}{\left(\omega_\Phi(\vec{p}+\vec{q}) - \omega_\Phi(\vec{p})\right)}$$

For $z_0 = \omega_\Phi(\vec{p}) - i\varepsilon$,

$$Res(f[z_0])$$
$$= \frac{1}{(\omega_\Phi(\vec{p}) - \omega_\Phi(\vec{p}+\vec{q}))} \frac{1}{(\omega_\Phi(\vec{p}) + \omega_\Phi(\vec{p}+\vec{q}))} \frac{\omega_\Phi(\vec{p})}{(\omega_\Phi(\vec{p}) + \Delta\mathcal{E}_\alpha)} \frac{1}{(2\omega_\Phi(\vec{p}))}$$
$$= \frac{1}{2} \frac{1}{(\omega_\Phi(\vec{p}) - \omega_\Phi(\vec{p}+\vec{q}))} \frac{1}{(\omega_\Phi(\vec{p}) + \omega_\Phi(\vec{p}+\vec{q}))} \frac{1}{(\omega_\Phi(\vec{p}) + \Delta\mathcal{E}_\alpha)}.$$

And then we obtain,

$$-2\pi i \sum_{z_0} Res(f[z_0])$$
$$= \frac{\pi i}{(\omega_\Phi(\vec{p}) + \omega_\Phi(\vec{p}+\vec{q}))(\omega_\Phi(\vec{p}) + \Delta\mathcal{E}_\alpha)(\omega_\Phi(\vec{p}+\vec{q}) + \Delta\mathcal{E}_\alpha)}.$$

We expand the term $(\vec{p}+\vec{q})^2$and relate them with the four-momentum transfer in the Breit frame Q^2 and the θ , angle between $\vec{p}$ and $\vec{q}$,

$$(\vec{p}+\vec{q})^2 = \vec{p}^2 + \vec{q}^2 + 2\vec{p}\cdot\vec{q} = \vec{p}^2 + Q^2 + 2|\vec{p}|\sqrt{Q^2}cos\theta$$
$$= p^2 + Q^2 + 2p\sqrt{Q^2}x$$

Where $\vec{p}^2 = p^2$, $|\vec{p}| = p$ and $x = cos\theta$.

From the above results, the $G_E^N(Q^2)|_{\alpha,MC}$ becomes,

$$G_E^N(Q^2)|_{\alpha,MC}$$
$$= -\frac{2}{(2\pi)^4F^2}\langle N\uparrow| \int d^3p \times \int d^3x_1\, \bar{u}_0(\vec{x}_1) i\gamma^5 S(x_1) u_\alpha(\vec{x}_1) e^{i(\vec{p}+\vec{q})\cdot\vec{x}_1}$$
$$\times \int d^3x_2 \bar{u}_\alpha(\vec{x}_2) i\gamma^5 S(x_2) u_0(\vec{x}_2)\, e^{-i\vec{p}\cdot\vec{x}_2} \times \left(f_{3ij} + \frac{f_{8ij}}{\sqrt{3}}\right)\lambda_i\lambda_j$$
$$\times \frac{\pi i}{(\omega_\Phi(\vec{p}) + \omega_\Phi(\vec{p}+\vec{q}))(\omega_\Phi(\vec{p}) + \Delta\mathcal{E}_\alpha)(\omega_\Phi(\vec{p}+\vec{q}) + \Delta\mathcal{E}_\alpha)}|N\uparrow\rangle.$$

Where $\omega_\Phi(\vec{p}+\vec{q})$, and $\omega_\Phi(\vec{p})$ are as below,

$$\omega_\Phi(\vec{p}+\vec{q}) \equiv \sqrt{M_\Phi^2 + (\vec{p}+\vec{q})^2} = \sqrt{M_\Phi^2 + p^2 + Q^2 + 2p\sqrt{Q^2}x}$$

$$\omega_\Phi(\vec{p}) \equiv \sqrt{M_\Phi^2 + \vec{p}^2} = \sqrt{M_\Phi^2 + p^2}.$$

The evaluation of $G_E^N(Q^2)|_{\alpha\beta,VC}$

$$\chi_{N_{s'}}^\dagger \chi_{N_s} G_E^N(Q^2)|_{\alpha\beta,VC}$$
$$= \left\langle \phi_0 \middle| \sum_{n=0}^2 \frac{i^n}{n!} \int \delta(t) d^4x d^4x_1 \ldots d^4x_n e^{-iq\cdot x} T[\mathcal{L}_r^{str}(x_1) \ldots \mathcal{L}_r^{str}(x_n) j_r^0(x)] \middle| \phi_0 \right\rangle_c^N$$

$$= 2\langle\phi_0| -\frac{1}{2}\int \delta(t) d^4x d^4x_1 d^4x_2 e^{-iq\cdot x} : \left(-\bar{\psi}(x_1) i\gamma^5 \frac{\lambda_k}{F} \Phi_i(x_1) S(r) \psi_\alpha(x_1)\right)$$

$$\times \left(Q\bar{\psi}_\alpha(x)\gamma^0\psi_\beta(x)\right)\left(-\bar{\psi}_\beta(x_2) i\gamma^5 \frac{\lambda_l}{F}\Phi_j(x_2) S(r)\psi(x_2)\right) : |\phi_0\rangle$$

$$= -\frac{1}{F^2}\langle\phi_0| \int \delta(t) d^4x d^4x_1 d^4x_2 e^{-iq\cdot x}\left[\bar{u}_0(\vec{x}_1) e^{i\mathcal{E}_0 t_1} i\gamma^5 \lambda_i S(x_1) u_\alpha(\vec{x}_1) e^{-i\mathcal{E}_\alpha t_1}\right]$$

$$\times \left[\theta(t_1 - t)\bar{u}_\alpha(\vec{x}) e^{i\mathcal{E}_\alpha t}\,(Q\,\gamma^0\,) u_\beta(\vec{x}) e^{-i\mathcal{E}_\beta t}\,\right] \times \theta(t - t_2)\bar{u}_\beta(\vec{x}_2) e^{i\mathcal{E}_\beta t_2}$$

$$\times\, i\gamma^5 \lambda_j S(x_2)\, u_0(\vec{x}_2) e^{-i\mathcal{E}_0 t_2} \times \delta_{ij}\int \frac{d^4p}{i(2\pi)^4}\frac{e^{-ip(x_1 - x_2)}}{M_\Phi^2 - p^2 - i\varepsilon}|\phi_0\rangle.$$

Consider the terms concerning with t , we obtain,

$$\int dt\, \delta(t)\, e^{-iq_0 t} e^{i\mathcal{E}_\alpha t} e^{-i\mathcal{E}_\beta t}\theta(t_1 - t)\,\theta(t - t_2) = \theta(t_1)\theta(-t_2).$$

We use the above result, then we get,

$$G_E^N(Q^2)|_{\alpha\beta,VC}$$

$$= \frac{i}{(2\pi)^4 F^2}\langle\phi_0| \int dt_1 dt_2 d^4p\,\left[\iint d^3x_1\, \bar{u}_0(\vec{x}_1) i\gamma^5 \lambda_i S(x_1) u_\alpha(\vec{x}_1) e^{i\vec{p}\cdot\vec{x}_1}\right]$$

$$\times \left[\int d^3x\ \bar{u}_\alpha(\vec{x})\,(Q\,\gamma^0\,) u_\beta(\vec{x})\, e^{i\vec{q}\cdot\vec{x}}\right] \times e^{-i(\mathcal{E}_\alpha - \mathcal{E}_0)t_1}\theta(t_1)$$

$$\times \left[\iint d^3x_2\, \bar{u}_\beta(\vec{x}_2) i\gamma^5 \lambda_i S(x_2) u_0(\vec{x}_2) e^{-i\vec{p}\cdot\vec{x}_2}\right] \times e^{i(\mathcal{E}_\beta - \mathcal{E}_0)t_2}\theta(-t_2)$$

$$\times \frac{e^{-ip_0(t_1 - t_2)}}{M_\Phi^2 + \vec{p}^2 - p_0^2 - i\varepsilon}|\phi_0\rangle.$$

We make use of the definition of $\theta(t)$ and then we obtain,

$$\int dt_1\, e^{-i\Delta\mathcal{E}_\alpha t_1}\,\theta(t_1)\quad e^{-ip_0 t_1} = \lim_{\eta\to 0}\frac{-i}{p_0 + \Delta\mathcal{E}_\alpha - i\eta},$$

$$\int dt_2\ \ e^{i\Delta\mathcal{E}_\beta t_2}\,\theta(-t_2)\, e^{ip_0 t_2} = \lim_{\eta\to 0}\frac{-i}{p_0 + \Delta\mathcal{E}_\beta - i\eta}.$$

Then the $G_E^N(Q^2)|_{\alpha\beta,VC}$ becomes,

$$G_E^N(Q^2)|_{\alpha\beta,VC}$$

$$= \frac{i}{(2\pi)^4 F^2}\langle\phi_0| \int d^3p\quad \left[\iint d^3x_1\, \bar{u}_0(\vec{x}_1) i\gamma^5 \lambda_i S(x_1) u_\alpha(\vec{x}_1) e^{i\vec{p}\cdot\vec{x}_1}\right]$$

$$\times \left[\int d^3x\ \bar{u}_\alpha(\vec{x})\,(Q\,\gamma^0\,) u_\beta(\vec{x})\, e^{i\vec{q}\cdot\vec{x}}\right]\left[\iint d^3x_2\, \bar{u}_\beta(\vec{x}_2) i\gamma^5 \lambda_i S(x_2) u_0(\vec{x}_2) e^{-i\vec{p}\cdot\vec{x}_2}\right]$$

$$\times \int dp_0\, \frac{1}{(p_0 + \Delta\mathcal{E}_\alpha) - i\eta}\frac{1}{(p_0 + \Delta\mathcal{E}_\beta) - i\eta}\frac{1}{(p_0 - \omega_\Phi(\vec{p}) + i\varepsilon)}\,\frac{1}{(p_0 + \omega_\Phi(\vec{p}) - i\varepsilon)}|\phi_0\rangle$$

We evaluate the term $\int dp_0$ by using the Residues Theorem, and then,

$$\int dp_0 \frac{1}{(p_0+\Delta\mathcal{E}_\alpha)-i\eta}\frac{1}{\left(p_0+\Delta\mathcal{E}_\beta\right)-i\eta}\frac{1}{(p_0-\omega_\Phi(\vec{p})+i\varepsilon)}\frac{1}{(p_0+\omega_\Phi(\vec{p})-i\varepsilon)}$$
$$= -2\pi i \sum_{z_0} Res(f[z_0])$$

There are four poles, three poles are in the upper plain, and one pole stay in the lower plane. In our calculation, we use the lower plane pole for simplicity.

For $z_0 = \omega_\Phi(\vec{p})$,

$$-2\pi i \sum_{z_0} Res(f[z_0])$$
$$= -2\pi i \frac{1}{(\omega_\Phi(\vec{p})+\Delta\mathcal{E}_\alpha)}\frac{1}{\left(\omega_\Phi(\vec{p})+\Delta\mathcal{E}_\beta\right)}\frac{1}{\left(2\omega_\Phi(\vec{p})\right)}$$
$$= \frac{-\pi i}{(\omega_\Phi(\vec{p})+\Delta\mathcal{E}_\alpha)\left(\omega_\Phi(\vec{p})+\Delta\mathcal{E}_\beta\right)\omega_\Phi(\vec{p})}.$$

We restrict our calculation to nucleons with spin up. Finally, the $G_E^N(Q^2)|_{\alpha\beta,VC}$ becomes,

$$G_E^N(Q^2)|_{\alpha\beta,VC}$$
$$= \frac{\pi}{(2\pi)^4F^2}\langle N\uparrow|\left|\int d^3p \;\left[\int d^3x_1\, \bar{u}_0(\vec{x}_1) i\gamma^5 S(x_1) u_\alpha(\vec{x}_1) e^{i\vec{p}\cdot\vec{x}_1}\right]\right.$$
$$\times \left[\int d^3x\; \bar{u}_\alpha(\vec{x})\,(Q\,\gamma^0\,) u_\beta(\vec{x})\; e^{i\vec{q}\cdot\vec{x}}\right]\left[\int d^3x_2\, \bar{u}_\beta(\vec{x}_2) i\gamma^5 S(x_2) u_0(\vec{x}_2) e^{-i\vec{p}\cdot\vec{x}_2}\right]$$
$$\times \frac{\lambda_i\lambda_i}{(\omega_\Phi(\vec{p})+\Delta\mathcal{E}_\alpha)\left(\omega_\Phi(\vec{p})+\Delta\mathcal{E}_\beta\right)\omega_\Phi(\vec{p})}|N\uparrow\rangle.$$

6) Evaluation of $\int d^3p\, \vec{p}\, \left(\vec{\sigma}\cdot\frac{\vec{p}+\vec{q}}{|\vec{p}+\vec{q}|}\right)\left(\vec{\sigma}\cdot\frac{\vec{p}}{|\vec{p}|}\right)$

$$= \int_0^\infty dp p^2 \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi\;\vec{p}\,\left(\vec{\sigma}\cdot\frac{\vec{p}+\vec{q}}{|\vec{p}+\vec{q}|}\right)\left(\vec{\sigma}\cdot\frac{\vec{p}}{|\vec{p}|}\right)$$
$$= \int_0^\infty dp \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi\;\frac{p^2}{|\vec{p}+\vec{q}|}\,\hat{p}\,\left(\vec{\sigma}\cdot(\vec{p}+\vec{q})\right)(\vec{\sigma}\cdot\vec{p}).$$

We use the identity of

$$(\vec{\sigma}\cdot\vec{A})(\vec{\sigma}\cdot\vec{B}) = \vec{A}\cdot\vec{B} + i\vec{\sigma}\cdot(\vec{A}\times\vec{B}).$$

And then, we get,

$$= \int_0^\infty dp \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi\;\frac{p^2}{|\vec{p}+\vec{q}|}\,\hat{p}\,\left[\vec{p}\cdot(\vec{p}+\vec{q}) + i\vec{\sigma}\right.$$
$$\left.\cdot\left((\vec{p}+\vec{q})\times\vec{p}\right)\right].$$

We proceed by evaluating the below term first,

$$\int_0^{2\pi} d\phi\ \ \hat{p}\left[\vec{p}\cdot(\vec{p}+\vec{q})+i\vec{\sigma}\cdot\big((\vec{p}+\vec{q})\times\vec{p}\big)\right]$$

The unit momentum vector $\hat{p}$ and the spin operator $\vec{\sigma}$ with their components in space can be shown as below,

$$\hat{p}\ = \sin\theta\cos\phi\ \hat{x} + \ \sin\theta\sin\phi\ \hat{y} + \ \cos\theta\ \hat{z}$$
$$\hat{\sigma}\ = \sigma_x\ \hat{x} + \sigma_y\ \hat{y} + \sigma_z\ \hat{z}.$$

And by defining the $\vec{q}$ to be $\vec{q} = (0,0,q) = q\hat{z}$, we obtain,

$$\vec{p}\ \cdot(\vec{p}+\vec{q}) = \vec{p}^2 + p\sqrt{Q^2}\cos\theta.$$

and

$$\vec{\sigma}\ \cdot\big((\vec{p}+\vec{q})\times\vec{p}\big) = \vec{\sigma}\ \cdot(\vec{q}\times\vec{p}),$$
$$= \big(\sigma_x\ \hat{x}+\sigma_y\ \hat{y}+\sigma_z\ \hat{z}\big)\cdot(-qp\sin\theta\sin\phi\ \hat{x} + qp\sin\theta\cos\phi\ \hat{y}\ )\ ,$$
$$= -\sigma_x qp\sin\theta\sin\phi + \sigma_y\ qp\sin\theta\cos\phi.$$
$$\textstyle\int_0^{2\pi} d\phi\ \ \hat{p}\left[\vec{p}\ \cdot(\vec{p}+\vec{q})+i\vec{\sigma}\ \cdot\big((\vec{p}+\vec{q})\times\vec{p}\big)\right]$$
$$= \hat{x}i\sigma_y\ qp\sin^2\theta\textstyle\int_0^{2\pi} d\phi\cos^2\phi - \hat{y}i\sigma_x qp\sin^2\theta\int_0^{2\pi} d\phi\sin^2\phi\ .$$
$$+\hat{z}\ \left(p^2+p\sqrt{Q^2}\cos\theta\right)\cos\theta\textstyle\int_0^{2\pi} d\phi\ ,$$
$$= i\pi\ qp\sin^2\theta\ \left(\sigma_y\hat{x}-\sigma_x\hat{y}\right) + \hat{z}\ 2\pi\ \left(p^2+p\sqrt{Q^2}\cos\theta\right)\cos\theta.$$

From above, we have assigned $\vec{q} = q\hat{z}$, so we can conclude that,

$$\sigma_y\hat{x} - \sigma_x\hat{y} = \vec{\sigma}\times\hat{q}.$$

And re-write the right hand side of the equation to be as below,

$$= i\pi\ qp\sin^2\theta\ \ (\vec{\sigma}\times\hat{q}) + \left\{2\pi\ \left(p^2+p\sqrt{Q^2}\cos\theta\right)\cos\theta\right\}\ \hat{q}\ .$$

which has two terms, the term in $\hat{q}$ direction and $\vec{\sigma}\times\hat{q}$ direction.

$$\textstyle\int_0^{\infty} dpp^2\int_0^{\pi}\sin\theta\, d\theta\int_0^{2\pi} d\phi\ \ \vec{p}\ \left(\vec{\sigma}\ \cdot\frac{\vec{p}+\vec{q}}{|\vec{p}+\vec{q}|}\right)\left(\vec{\sigma}\ \cdot\frac{\vec{p}}{|\vec{p}|}\right)$$
$$= \int_0^{\infty} dp\frac{p^2}{|\vec{p}+\vec{q}|}\int_{-1}^{1} dx\left\{i\pi\ qp(1-x^2)\ (\vec{\sigma}\times\hat{q}) + 2\pi\ x\left(p^2+p\sqrt{Q^2}x\right)\ \hat{q}\ \right\}$$
$$= \int_0^{\infty} dpp^3\left(p^2+Q^2+2p\sqrt{Q^2}x\right)^{-1/2}\int_{-1}^{1} dx\left\{i\pi\ q(1-x^2)\ (\vec{\sigma}\times\hat{q})\right.$$
$$\left. + \ 2\pi\ x\left(p+\sqrt{Q^2}x\right)\ \hat{q}\ \right\}.$$

Appendix F

The notation and the upper and lower components of the quark wave functions in the 3rd, the 4th and the 5th excited states

The notation and the upper and lower components of the quark wave functions in the 3rd, the 4th and the 5th excited states

TABLE 9 quark wave functions in the 3rd, the 4th and the 5th excited states

State α (u_α)	Label	$g\alpha(r)$ and $f\alpha(r)$ With $R\alpha$ and $\rho\alpha$
u_6	$2p_{1/2}$	$g6r = \frac{e^{-\frac{r^2}{2R6^2}} r(\frac{5}{2} - \frac{r^2}{R6^2})}{R6}$
		$f6r = -e^{-\frac{r^2}{2R6^2}} \left(\frac{15}{2} + \frac{r^4}{R6^4} - \frac{15r^2}{2R6^2}\right) \rho 6$
u_7	$2p_{3/2}$	$g7r = \frac{e^{-\frac{r^2}{2R7^2}} r(\frac{5}{2} - \frac{r^2}{R7^2})}{R7}$
		$f7r = \frac{e^{-\frac{r^2}{2R7^2}} r^2 \left(\frac{9}{2} - \frac{r^2}{R7^2}\right) \rho 7}{R7^2}$
u_8	$1f_{5/2}$	$g8r = \frac{e^{-\frac{r^2}{2R8^2}} r^3}{R8^3}$
		$f8r = -\frac{e^{-\frac{r^2}{2R8^2}} r^2 \left(7 - \frac{r^2}{R8^2}\right) \rho 8}{R8^2}$

Table 9 (Continued)

State α (u_α)	Label	$g\alpha(r)$ and $f\alpha(r)$ With $R\alpha$ and $\rho\alpha$
u_9	$1f_{7/2}$	$g9r = \frac{e^{-\frac{r^2}{2R9^2}} r^3}{R9^3}$
		$f9r = \frac{e^{-\frac{r^2}{2R9^2}} r^4 \rho 9}{R9^4}$
u_{10}	$1g_{7/2}$	$g10r = \frac{e^{-\frac{r^2}{2R10^2}} r^4}{R10^4}$
		$f10r = -\frac{e^{-\frac{r^2}{2R10^2}} r^3 \left(9 - \frac{r^2}{R10^2}\right) \rho 10}{R10^3}$
u_{11}	$1g_{9/2}$	$g11r = \frac{e^{-\frac{r^2}{2R11^2}} r^4}{R11^4}$
		$f11r = \frac{e^{-\frac{r^2}{2R11^2}} r^5 rho11}{R11^5}$
u_{12}	$2d_{3/2}$	$g12r = \frac{e^{-\frac{r^2}{2R12^2}} r^2 (\frac{7}{2} - \frac{r^2}{R12^2})}{R12^2}$
		$f12r = -\frac{e^{-\frac{r^2}{2R12^2}} r \left(\frac{35}{2} + \frac{r^4}{R12^4} - \frac{21r^2}{2R12^2}\right) \rho 12}{R12}$
u_{13}	$2d_{5/2}$	$g13r = \frac{e^{-\frac{r^2}{2R13^2}} r^2 (\frac{7}{2} - \frac{r^2}{R13^2})}{R13^2}$
		$f13r = \frac{e^{-\frac{r^2}{2R13^2}} r^3 \left(\frac{9}{2} - \frac{r^2}{R13^2}\right) \rho 13}{R13^3}$

Table 9 (Continued)

State α (u_α)	Label	$g\alpha(r)$ and $f\alpha(r)$ With $R\alpha$ and $\rho\alpha$
u_{14}	$3s_{1/2}$	$g14r = \frac{e^{-\frac{r^2}{2R14^2}}(4r^4 - 20r^2R14^2 + 15R14^4)}{8R14^4}$
		$f14r = \frac{e^{-\frac{r^2}{2R14^2}}r\left(\frac{35}{8} + \frac{r^4}{2R14^4} - \frac{7r^2}{2R14^2}\right)\rho14}{R14}$
u_{15}	$1h_{9/2}$	$g15r = \frac{e^{-\frac{r^2}{2R15^2}}r^5}{R15^5}$
		$f15r = -\frac{e^{-\frac{r^2}{2R15^2}}r^4\left(11 - \frac{r^2}{R15^2}\right)\rho15}{R15^4}$
u_{16}	$1h_{11/2}$	$g16r = \frac{e^{-\frac{r^2}{2R16^2}}r^5}{R16^5}$
		$f16r = \frac{e^{-\frac{r^2}{2R16^2}}r^6\rho16}{R16^6}$
u_{17}	$2f_{5/2}$	$g17r = \frac{e^{-\frac{r^2}{2R17^2}}r^3\left(\frac{9}{2} - \frac{r^2}{R17^2}\right)}{R17^3}$
		$f17r = -\frac{e^{-\frac{r^2}{2R17^2}}r^2\left(\frac{63}{2} + \frac{r^4}{R17^4} - \frac{27r^2}{2R17^2}\right)\rho17}{R17^2}$
u_{18}	$2f_{7/2}$	$g18r = \frac{e^{-\frac{r^2}{2R18^2}}r^3\left(\frac{9}{2} - \frac{r^2}{R18^2}\right)}{R18^3}$
		$f18r = \frac{e^{-\frac{r^2}{2R18^2}}r^4\left(\frac{11}{2} - \frac{r^2}{R18^2}\right)\rho18}{R18^4}$

Table 9 (Continued)

State α (u_α)	Label	$g\alpha(r)$ and $f\alpha(r)$ With $R\alpha$ and $\rho\alpha$
u_{19}	$3p_{1/2}$	$g19r = \frac{e^{-\frac{r^2}{2R19^2}} r(4r^4 - 28r^2R19^2 + 35R19^4)}{8R19^5}$
		$f19r = -e^{-\frac{r^2}{2R19^2}}\left(\frac{105}{8} - \frac{r^6}{2R19^6} + \frac{7r^4}{R19^4} - \frac{175r^2}{8R19^2}\right)\rho 19$
u_{20}	$3p_{3/2}$	$g20r = \frac{e^{-\frac{r^2}{2R20^2}} r(4r^4 - 28r^2R20^2 + 35R20^4)}{8R20^5}$
		$f20r = \frac{e^{-\frac{r^2}{2R20^2}} r^2\left(\frac{63}{8} + \frac{r^4}{2R20^4} - \frac{9r^2}{2R20^2}\right)\rho 20}{R20^2}$

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